Computational meshless methods for engineering

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Background
Mechanical and civil engineering involves many problems in which predictions are made of movements & deformations under loading, temperature etc. using computational models. The most popular of these is the Finite Element Method, which has been around since the 1970s. It works by splitting a problem up into discrete elements (a “mesh”), joined together at points or nodes. Each element has simple rules for its behaviour. The combination of many elements produces a model which can approach the complex and unknown actual behaviour of the item being modelled. However, once we wish to model in 3D the meshing process itself becomes a major part of the computational effort. A further problem with the finite element method is that it stops working if deformations distort the mesh. Meshless methods offer a solution. As the name suggests there is no mesh of elements. An object is modelled with a set of nodes alone. At Durham we have been developing meshless methods for engineering since 2004. Below are some recent highlights.

Improving accuracy
In meshless methods, the nodes distributed inside a problem domain each have influence over a small region of the domain surrounding (the nodal support) This influence diminishes with distance from the node in a smooth fashion. We have shown that inaccuracies found with meshless methods can arise due to the topological arrangement of nodes and the choice of rule for the determination of the zone of support. Prior to this, the inaccuracies were tackled using different mathematical formulations for the underlying behaviour, but we have shown this to be not important. We have developed a hierarchical scheme for the nodes which avoids the inaccuracy associated with nodal layout [1].

Modelling fracture
Cracks in the real world are three-dimensional. Modelling cracks in 3D remains a challenging topic in computational solid mechanics and much of the research in the past has been in 2D since the geometry of crack surfaces can be difficult both to describe and to capture as cracks evolve. This is unlike the case in 2D where cracks can be represented as combinations of lines or curves. To overcome this problem, we have developed a numerical framework coupling a meshless method (the element-free Galerkin method, EFGM) and the level set method (LSM) for 3D crack modelling. In this framework, the EFGM analyzes the mechanical behaviour of the crack and LSM accurately describes and captures the crack geometry as it Propagates [2].

In this framework, the crack motion is tracked by the LSM while the EFGM undertakes the stress analysis based on the level sets. For each crack surface two level sets are used to describe the geometry; one measures signed distance normal to the crack surface and the other measures signed distance tangential to the crack front. They together form a local curvilinear coordinate system in the vicinity of the crack front. A number of tests have been carried out, including penny-shaped and elliptic cracks (corresponding to flaws in materials found in the real world) subjected to various boundary conditions which show the effectiveness of the developed framework.