Sky Projected Shack-Hartmann Laser Guide Star

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ABSTRACT

We describe SPLASH (Sky Projected Laser Array Shack-Hartmann) which is a method of laser guide star (LGS) wavefront sensing with reduced focal anisoplanatism (FA). We present the results of a semi-theoretical analysis and a semi-geometrical simulation of SPLASH, allowing a direct comparison between SPLASH and a conventional laser guide star system. We show that SPLASH is significantly less susceptible to focal anisoplanatism than a conventional LGS.

Keywords: Adaptive optics, Laser guide stars, ELTs

1. INTRODUCTION

Focal anisoplanatism (FA) is a limitation of the current generation of laser guide star adaptive optics (LGS AO) systems and will render a single LGS AO system on an extremely large telescope (ELT) virtually unusable. Multiconjugate adaptive optics (MCAO) will mitigate some of the problems by essentially combining data from different lasers, however a LGS system without focal anisoplanatism would be a huge bonus with or without MCAO. Here we outline a possible alternative implementation of a LGS in which the atmosphere is sensed on the upward path of the laser through the atmosphere. SPLASH, or Sky Projected Laser Array Shack-Hartmann\textsuperscript{1} is a pseudo-reverse of the usual method of Shack-Hartmann wavefront sensing. An array of converging beams is launched monostatically from the telescope to produce an array of Shack-Hartmann spots projected onto the sky, which are then imaged by the whole telescope.

A schematic outline of a SPLASH is shown in Fig. 1. An array of converging beams is focused on the sodium layer, conceptually the same as placing a huge lenslet array over the top of the telescope. The system could also be implemented with a Rayleigh laser. The beam size, at the telescope, is $\sim r_0$ as in a conventional Shack-Hartmann system and therefore the focal spot quality will be largely unaffected by the atmosphere but the position will be shifted due to the local wavefront gradient. On the downward passage of light through the atmosphere the light is collected by the whole telescope and conventionally imaged without any kind of wavefront sensor. The array image will be both distorted and shifted due to the global tip and tilt. The relative positions of the spot images will therefore be a measure of the local tilt minus the global tilt — i.e. exactly the same quantity as measured in a conventional LGS with a Shack-Hartmann wavefront sensor (WFS). The major advantage of this system is that the cone effect will be much reduced. The system proposed in Fig. 1 appears rather similar to the concept of stitching and butting (see, e.g., Fried,\textsuperscript{2} Parenti and Sasiela\textsuperscript{3}) whereby an array of LGSs is used in order to reduce focal anisoplanatism. However, in this approach the atmosphere is still sensed on the return path, and the light is sent to a wavefront sensor. Problems then occur in trying to join the resulting phase maps together.

There are a number of issues associated with SPLASH wavefront sensing.

1. A major one is the fact that Fig. 1 is drawn assuming geometrical optics whereas in reality the beams will diffract. Assuming no aberrations, a 10cm sodium beam focused at 90km has a full beam width of 1.3m, which is clearly much too large for the system to work. Furthermore, the spots will further merge due to the actual displacements due to the atmosphere, which will be of a similar magnitude. There are two potential solutions to this problem:

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Figure 1. Concept of SPLASH, showing the upward passage of the beams. We show a possible optical implementation whereby the laser is launched via a lenslet array. This is only a conceptual diagram and not a formal optical design. The size of each of the converging beams is \( \sim r_0 \) although we have only shown 4 here for clarity. Furthermore, we have shown the beams as converging to a spot, whereas in reality they would be diffracted. See text for more discussion.

(a) An assumed aperture size of 10cm is (currently) rather extreme. For infrared observations an aperture size closer to around 50cm would be more realistic, leading to a total spot width on the atmosphere of 26cm.

(b) If smaller subapertures are required then one can envisage time sequencing each subaperture so that they do not overlap on the sky.

2. The system will still suffer from some focal anisoplanatism, as can be seen in Fig. 1. The laser samples smaller areas in the atmosphere with increasing height. The problems with diffraction, described above, may actually tend to mitigate this.

3. There is a further focal anisoplanatic effect in that the magnitude of the spot displacement is dependent on the turbulence layer height. There is a large lever-arm effect so that the technique is more sensitive to lower layer turbulence. For a sodium beacon this should not be a serious limitation.

4. The image projected onto the sky must subtend an angle less than the isokinetic patch (the angle over which the wavefront tilt can be considered to be isoplanatic).

5. For larger telescope apertures, the effects of turbulence on the return path of the light must be considered. For aperture diameters greater than \( \sim 10r_0 \), speckle will be observed in the spot images. As the image becomes increasingly speckled, the spot centroids will become less correlated with the local phase gradient. This effect can be overcome by imaging the spots through a return subaperture that is several times larger than the laser launch subapertures but smaller than the full telescope aperture. While only one is required
for the system to work, it would be desirable to use several return subapertures and combine the wavefront data from them in order to make full use of the returned light and maximise the signal-noise ratio.

Against these problems SPLASH has the advantages of reduced focal anisoplanatism and, when using a sodium laser, reduced laser power density on the sodium layer.

2. THEORETICAL ESTIMATE OF THE EFFECTS OF FOCAL ANISOPLANATISM IN SPLASH

The effects of focal anisoplanatism on SPLASH were considered in terms of modal correction of Kolmogorov turbulence using Zernike polynomials as the basis set. The phase distortion, \( \phi(Rr) \), across a circular aperture can be expressed in terms of Zernike modes, \( Z_j \), as

\[
\phi(Rr) = \sum_{j=1}^{\infty} a_j Z_j(r)
\]

where \( R \) is the radius of the aperture, \( r \) is the coordinate normalised to unit radius and \( a_j \) are the Zernike coefficients, given by

\[
a_j = \int dr \phi(Rr) Z_j(r) W(r),
\]

where \( W(r) \) is the pupil function. We explicitly exclude the aperture-averaged phase (piston) from the summation in equation 1. If the first \( N \) Zernike modes could be perfectly corrected, the residual wavefront distortion would then be

\[
\phi(Rr) = \sum_{j=1}^{N} a_j Z_j(r).
\]

The mean square residual error can therefore be written as

\[
\sigma^2 = \langle \phi^2 \rangle - \sum_{j=1}^{N} \langle |a_j|^2 \rangle.
\]

A wavefront sensor gives estimates \( b_j \) of the coefficients \( a_j \) which differ by a noise term,

\[
b_j = a_j + e_j,
\]

where \( e_j \) represents noise due to various sources, but here we assume the only source of noise to be focal anisoplanatism. Assuming this is the only error in the system (i.e. the wavefront error can be perfectly corrected to the degree which it is sensed) the residual wavefront distortion is then

\[
\phi(Rr) = \sum_{j=1}^{\infty} (a_j - b_j) Z_j(r).
\]

Hence, with reference to equation 4, the residual phase variance for a system with imperfect sensing but perfect correction of \( N \) Zernike modes is

\[
\sigma_{\phi}^2 = \sum_{j=1}^{N} \langle (a_j - b_j)(a_j - b_j) \rangle = \sum_{j=1}^{\infty} \left( \langle a_j a_j \rangle + \langle b_j b_j \rangle - \langle a_j b_j \rangle - \langle b_j a_j \rangle \right).
\]

If we assume that \( \langle a \rangle = \langle b \rangle \), the residual phase variance for a system correcting \( N \) Zernike modes is

\[
\sigma_{\phi,N}^2 = \sum_{j=1}^{N} 2 \left( \langle a_j^2 \rangle - \langle a_j b_j \rangle \right) + \sum_{j=N+1}^{\infty} \langle a_j^2 \rangle.
\]
If we now make the assumption that the noise term due to focal anisoplanatism \( e_j \) is independent of the Zernike coefficients \( a_j \), the equation for \( N \) corrected modes becomes

\[
\sigma_{\phi, N}^2 = \sum_{j=1}^{N} \langle e_j^2 \rangle + \sum_{j=N+1}^{\infty} \langle a_j^2 \rangle .
\]

(9)

We estimate the noise term, \( e_j \), using a variation on Wilson & Jenkins’ treatment of wavefront sensor noise errors.\(^5\) In this treatment, the estimated modal coefficients from a Shack-Hartmann wavefront sensor are given by

\[
b = \frac{\pi D}{\lambda} D^{-1}(g + \delta g),
\]

(10)

where \( b \) is the vector of subaperture tilts and \( D^{-1} \) is the generalised inverse of the matrix of subaperture-averaged derivatives of the basis functions (in this case Zernike modes). \( \delta g \) is the vector of subaperture slope errors which we take to be entirely due to focal anisoplanatism – we ignore other sources of error. The covariance matrix of input and measured modal coefficients is given by

\[
C_{i,j}^{WFS} = \left( \frac{\pi D}{\lambda} \right)^2 (DD^T)^{-1} \langle \delta g^2 \rangle = (DD^T)^{-1} \sigma_c^2,
\]

(11)

where \( \sigma_c^2 \) is the one-dimensional variance of the motion of the centroid of a stellar image seen through a square aperture of side \( d \), given by\(^6\)

\[
\sigma_c^2 = 0.162 \lambda^2 r_0^{-5/3} d^{-1/3}.
\]

(12)

Figure 2. Theoretical prediction of SPLASH performance as compared with an equivalent conventional LGS/Shack-Hartmann wavefront sensor system. Results show residual wavefront variance at a wavelength of 2.2 µm for a 4.2m telescope with an 8 × 8 array of subapertures, a laser wavelength of 500nm, a focus altitude of 10km, and a single atmospheric layer at 2.5km with \( r_0 = 15 \)cm and infinite outer scale.
The covariance matrix $C_{i,j}^{WFS}$ can then be used to calculate the residual wavefront variance due to focal anisoplanatism,

$$\langle e_j^2 \rangle = (1 - f) \times C_{jj}^{WFS}, \quad (13)$$

where $f$ is a subaperture tilt correlation factor. This factor is dependent on the turbulence altitude and is equal to the correlation between tilt coefficients measured for a source at infinity and coefficients measured for a beacon at a finite altitude. It is equal to 1 in the case where the observed centroid displacement is perfectly correlated with the tilt on the subaperture, i.e. when there is no focal anisoplanatism because the turbulence is all at ground level. For further explanation see Wilson & Jenkins.\(^5\)

Thus we have all the information required to predict the residual wavefront variance for a given number of corrected Zernike modes. We have assumed the cone geometry shown in Fig. 1, and that the system is capable of perfectly correcting Zernike modes to the degree they can be sensed. We also assume that tip and tilt across the entire telescope aperture can be perfectly sensed, since in reality these would not be sensed using the laser beacon.

The results of the theoretical analysis of SPLASH are shown in Fig. 2, for an AO system suitable for observing in the infrared. For comparison, the graph includes the equivalent results for a conventional laser guide star system and for a perfect wavefront sensor (with a Noll covariance matrix). The results indicate that a SPLASH system could be expected to perform significantly better than an equivalent system using a conventional laser guide star with a Shack-Hartmann wavefront sensor. However, it should be noted that this analysis does rely on the assumption that the noise due to FA on a given Zernike term is independent of the corresponding Zernike coefficient. This approach also ignores the effects of aberrations introduced on the downward path of the light from the focused spots.

### 3. SEMI-GEOMETRICAL SIMULATION OF SPLASH

A semi-geometrical Monte Carlo simulation of SPLASH has also been implemented. The simulation assumes the diffraction-free geometry illustrated in Fig. 1 but, unlike the theoretical performance estimate, it takes into account the return path of the light through the atmosphere.

The performance of SPLASH as a wavefront sensor is assessed by evaluating Zernike covariance matrices in the simulation and using these to calculate the residual wavefront variance for a given number of corrected modes. From equation 7, for $N$ corrected modes the residual phase variance is

$$\sigma_\phi^2 = \sum_{i=1}^\infty \langle a_i a_i \rangle + \sum_{i=1}^N \left( \langle b_i b_i \rangle - 2 \langle a_i b_i \rangle \right), \quad (14)$$

where $a_j a_j$ are the on-diagonal elements of the Noll covariance matrix and $b_j b_j$ and $a_j b_j$ are the on-diagonal elements of the covariance matrices evaluated in the simulation.

The simulation is implemented as follows. The atmosphere is modelled as a series of frozen moving Kolmogorov screens at different altitudes. The wavefront sensor itself is modelled in two stages. The upward passage of light from each subaperture through the atmosphere is modelled by projecting the portions of each phase screen intersected by the beam along the path down onto the subaperture as shown in Fig. 3. The phase contributions from each layer are summed and an FFT of the phase is used to form an image of the spot at the focus altitude. The downward path is modelled separately in a similar way but with a different geometry because the spot is imaged through the full telescope aperture, as shown in Fig. 3.

We make the assumption that the spot is isoplanatic, then the point spread function (PSF) from the upward path can be convolved with that from the downward path to form the final image of each spot. As each subaperture is simulated separately, the images of the spots have to be “stitched” together with the correct spacing if the effects of stray light from adjacent subapertures are to be considered. However, the results presented here do not include this step. It is assumed that, should it be necessary, it will be possible to time-interleave the spots to prevent them overlapping. We also assume that the laser is sufficiently bright to allow range gating such that spot elongation effects are negligible.
Figure 3. Left: The upward paths followed by the beams. Each beam samples the atmosphere above its own subaperture, and each beam is affected separately by focal anisoplanatism. Right: The downward paths taken by the light from each spot. The spots are observed through the full telescope aperture so the light does not pass through the same section of atmosphere as the upward-propagating (wavefront sensing) beams. The upward-propagating beams are affected by the local wavefront gradient, and the downward beams by the global gradient. The paths from one subaperture are darkened to clearly show the regions of atmosphere affecting light from that subaperture.

At each iteration of the simulation the phase distortion is numerically decomposed into Zernike modes as described by equation 2. The vector of SPLASH spot centroids are decomposed in the same way to obtain the modal coefficients as estimated by the wavefront sensor. Averaging over many phase screens, the matrix $a_i b_j$ is constructed from the covariance of the actual Zernike coefficients with the measured coefficients and the matrix $b_i b_j$ from the covariance of the measured coefficients with themselves. If the wavefront sensor was perfect, the covariance matrices would be identical to the Noll matrix, whereas for imperfect wavefront sensing the values on the diagonals of the matrices are smaller. The off-diagonal values are not discussed here as they are not required in this analysis.

As in the theoretical analysis, SPLASH measurements of tip and tilt are ignored and these modes are assumed to be perfectly sensed and corrected.

Fig. 4 shows the simulation results for a SPLASH system in which the spots are imaged through the full telescope, together with the theoretical predictions and reference results plotted in Fig. 2. The parameters chosen (telescope diameter 4.2m, $r_0 = 15cm$) place the simulated system in the regime in which significant speckling of the spot images is to be expected, leading to poor correlation between the spot centroids and subaperture tilts. As expected, the simulation performs considerably worse than predicted by theory (which does not take the downward path of the light into account) thus highlighting the necessity to use a smaller return aperture for larger telescopes.

Simulation results for a system with a smaller return aperture (half the diameter of the telescope aperture) are shown in Fig. 5. In this case the theoretical and simulated results for SPLASH agree well, and both sets clearly indicate better performance than a conventional laser guide star. Indeed, the results are closer to the “perfect” results (the best any wavefront sensor could possibly achieve) than to the conventional laser guide star results.
Figure 4. Results of theoretical analysis and semi-geometrical simulation of SPLASH on a 4.2m telescope with an $8 \times 8$ array of subapertures. The residual wavefront variance is shown at 2.2 $\mu$m with a laser wavelength of 500nm, a focus altitude of 10km, and a single atmospheric layer at 2.5km with $r_0 = 15$cm and infinite outer scale. The SPLASH laser spots are imaged through the whole telescope aperture.

Figure 5. Results of theoretical analysis and semi-geometrical simulation of SPLASH on a 4.2m telescope with an $8 \times 8$ array of subapertures. The residual wavefront variance is shown at 2.2 $\mu$m with a laser wavelength of 500nm, a focus altitude of 10km, and a single atmospheric layer at 2.5km with $r_0 = 15$cm and infinite outer scale. For imaging the SPLASH spots, the main telescope aperture is masked down to a 2.1m circular aperture to limit speckling of the spot images.
4. CONCLUSIONS

We have described a new method of laser guide star wavefront sensing in which an array of Shack-Hartmann spots are projected onto the sky and then imaged through the telescope. We have shown through theoretical calculations and simulations that this method suffers considerably less from focal anisoplanatism than a conventional laser guide star system.

We intend to carry out further simulations taking diffraction into account, and also to investigate the possibility of increasing signal-noise by imaging the laser spots through multiple return subapertures.

REFERENCES


