Reliability analysis of tunnel convergence: Is it worth the effort?

Faustino, G.*1, Bilé Serra, J.*2

1 Laboratório Nacional de Engenharia Civil, Lisbon, Portugal
   (presently at COBA Consulting Engineers)
2 Laboratório Nacional de Engenharia Civil, Lisbon, Portugal

ABSTRACT The safety of tunnel excavation is generally evaluated by means of deterministic approaches. The complexity of the phenomena mobilized by tunnel excavation has favored the adoption of simplified methods of analysis within which the convergence-confinement method is frequently preferred. This analytical method was used to perform a reliability analysis of the influence of two key design parameters, namely the distance lag at the support installation position and the stability limit of convergence through the analysis of a case study. Reliability analysis was performed using FORM methodologies in order to approximate the limit state functions. These functions represent the two main radial failure mechanisms, i.e. the resistance of the support, when the maximum pressure allowed is mobilized, and the excessive convergence of the cavity, which is assessed with limit values for deformation. The results are considered separately and together. The possibility of the applicability of finite element methods within this methodology is also investigated, using the software PLAXIS.

1 INTRODUCTION

The safety of tunnel excavation is typically evaluated by means of deterministic analyses. The complexity of the phenomena mobilized by tunnel excavation and the paucity of geological and geotechnical data has favored the adoption of simplified methods of analysis in the past.

From the geotechnical point of view conventional concepts such as the global safety factor are still used for the purpose of assessing the safety level. It is based on the experience of generations of both design and site engineers, having many times proved successful and led to adequate safety standards. However, a formal weakness of this concept is that the uncertainty is not associated with the real causes. Since the uncertainty and variability of the ground and the randomness of support properties play a significant role in the reliability level achieved with a given design, the issue of the applicability of probabilistic based methods in tunnel design is gaining relevance.

There have been several probabilistic analysis of underground excavation of stability problems (e.g. Li and Low, 2010 or Oreste, 2005). On the subject of tunnel support, Schweiger et al. (2001) carried out a probabilistic reliability analysis with a finite element code. Albeit less frequent, reliability evaluation of serviceability performance, e.g. subsidence, has also been addressed in the past (Mollon et al., 2009 and Serra and Miranda, 2013).

This paper addresses the reliability evaluation of different modes of failure of a deep tunnel by study the influence of both distance from the tunnel face of the support position and different limit failure values of the probability of failure. These calculations were performed using a simplified methodology to analyze the interaction between the ground and the support - the Convergence-Confinement Method – commonly
used in preliminary design phases. It was also investigated with a brief example if more representative analysis methods such as finite element methods (FEM) can be used to perform reliability analysis.

2 CONVERGENCE-CONFINEMENT METHOD

2.1 General considerations

The convergence-confinement (CC) method, i.e. the characteristic curve method, was developed to simulate three dimensional effects caused by the excavation of a tunnel by means of a simplified two dimensional analysis. Strictly speaking, the validity of this method is restricted to deep tunnels where the prevailing failure mechanism is related to the convergence of the cavity (Panet and Guellec, 1974). In an ingenious but simple way, these effects can be accounted for by considering a fictitious inner pressure at the tunnel wall, that in a plane deformation analysis would correspond to a given convergence value (Figure 1).

![Figure 1. Fictitious inner pressure on the support and confinement loss ratio (Martin, 2012)](image)

With this information it is possible to construct the longitudinal displacement profile (LDP) which aims at including the three dimensional effects in a simplified two dimensional analysis. It consists on a diagram that represents the radial displacement of the tunnel wall along the axis. In order to construct a rigorous analytical profile it is necessary to carry out a three-dimensional analysis. However, several analytical solutions to construct this curve are available. The one by Vlachopoulos and Diederich (2009) was used in this paper.

The convergence-confinement method requires the construction of three basic curves, addressed in detail in several references, such as in Lü et al. (2011). The first one is the longitudinal deformation profile which was already described. The other two will be addressed bellow.

2.2 Characteristic curves of the ground and of the liner

The ground reaction curve (GRC), or characteristic curve, defines the relationship between the inner pressure decrease from the geostatic condition and the radial displacement at the tunnel wall. As the tunnel face advances and moves away from the analysis section, the fictitious support pressure decreases. As a consequence, the displacement will gradually increase, as does the plastic radius, whereas the support pressure at equilibrium tends to reduce.

The vast majority of the closed form solutions assume elastic-plastic mechanical response with either Mohr-Coulomb or Hoek-Brown failure criterion. In this paper the Duncan Fama (1993) analytical solution, which is based on the Mohr-Coulomb failure criterion, was used to construct the GRC.

The support characteristic curve (SCC) defines the dependence between the pressure at the support system and its inward radial displacement. If one admits an elastic/perfectly-plastic response of the support, it takes only two parameters to fully describe it, i.e. (i) the elastic stiffness $k_s$, which is the ratio between the radial pressure increment and the variation of radial deformation, and (ii) the resistance of the support $P_{s_{\text{max}}}$, beyond which the support yields and fails. A common design criterion is to limit the maximum support load to a fraction of the resistance, that is, to define a limit value to the deformation.

The elastic stiffness and the limit pressure were determined following Hoek et al (2002).

2.3 Attaining equilibrium

As illustrated in Figure 2, one can simulate the interaction between the support and the ground and determine the equilibrium pressure ($P_{\text{eq}}$) and displacement ($u_{\text{eq}}$).

Prior to the support installation, a displacement $u_{\text{in}}$ (which can be determined by the $LDP$) has already occurred due to the confinement loss. The support is then installed and starts to deform until it attains both
equilibrium pressure and displacement compatibility with the ground at point A.

3 RELIABILITY ANALYSIS OF TUNNEL EXCAVATION

3.1 Level II reliability analysis

Level II methods are based on linear (1st order) or quadratic (2nd order) approximations of the limit state function (LSF). The former are named as FORM (First-Order Reliability Methods) (see Figure 3).

The probability of failure is quantified by means of a reliability index $\beta$, which corresponds to the distance between the center of the normalized space and the design point, which is the most probable point in case of failure. This is the point of first tangency in the space of the representative standard variables (e.g. $U_1$ and $U_2$ in Figure 3) between the origin centered circles and the limit state function.

$$\beta = \min \sqrt{\left[ \frac{x_i - \mu_i}{\sigma_i} \right]^T \left[ R \right]^{-1} \left[ \frac{x_i - \mu_i}{\sigma_i} \right]}$$

where $\mu_i$ are the mean and $\sigma_i$ the standard deviation of the random variables $x_i$. Equations (1) and (2) define the limit state functions:

$$g_1(X) = \frac{P_{s,\text{max}}}{P_{eq}} - 1 \quad [1]$$

$$g_2(X) = \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{eq}}} - \frac{\mu_{\varepsilon}}{R} \quad [2]$$

where $\varepsilon_{\text{max}}$ is the limit of the radial linear strain and $\varepsilon_{\text{eq}}$ stands for the inward radial displacement at equilibrium and the subscript eq stand for “in equilibrium”.

3.2 Limit state functions

Two performance functions were considered, concerning the structural resistance of the support ($g_1(X)$) and the maximum convergence allowed in the tunnel walls of radius $R$ ($g_2(X)$).

3.3 Computational implementation of the reliability analysis

The FORM methodology was implemented using an alternative interpretation of the Hasofer-Lind reliability index ($\beta_{HL}$), proposed by Low and Tang (2001). The methodology is described in detail in their work.

A continuous model of the ground by means of an elastic/perfectly-plastic deformation model obeying the Mohr-Coulomb failure criterion with isochoric deformation was assumed. The corresponding three random parameters are the deformability modulus ($E'$), the friction angle $\phi'$ and the apparent cohesion $c'$, expressed in terms of effective stress components all assumed to follow a Gaussian probability distribution with parameters $\mu_i$ and $\sigma_i$ and correlation matrix $R$. Regarding the support, the only discrete parameter is its resistance to radial pressure action.

The design point $x_d$, i.e. that on the limit state function with the smallest distance to the origin, is obtained by means of the minimization of the reliability index ($\beta$) as defined by Ditlevensen (1981).

The MS Excel solver add-in is used in a minimization problem of $\beta$ considering a few additional constraints, namely:

1) $g(X) = 0$, concerning the fact that the solution vector (design point) has to verify the condition of the limit state function
2) $P_{eq} \leq P_{s,\text{max}}$; and
3) $u_{\text{ground}} = u_{\text{support}}$

This process is addressed in detail in Faustino (2013).
4 A DEEP TUNNEL IN SOFT ROCK

4.1 Ground Properties

The example used to illustrate the reliability analysis of ground-support interaction (using the convergence-confinement method) consists of a circular tunnel of radius equal to 5 m under a hydrostatic stress field of 7 MPa.

The properties are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\mu_i$</th>
<th>$\sigma_i$</th>
<th>CoV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E'$ [MPa]</td>
<td>1800</td>
<td>500</td>
<td>28</td>
</tr>
<tr>
<td>$\phi'$ [deg]</td>
<td>23</td>
<td>3.3</td>
<td>22</td>
</tr>
<tr>
<td>$c'$ [kPa]</td>
<td>1500</td>
<td>320</td>
<td>14</td>
</tr>
</tbody>
</table>

The Poisson’s ratio of the rock mass is assumed as deterministic and equal to 0.3.

4.2 Support properties

The support is also regarded as deterministic. A shotcrete lining 25 cm thick is considered. The adopted mechanical properties are as follows: Young’s modulus $E=30$ GPa with an uniaxial compression strength of 35 MPa and a Poisson’s ration of $v=0.2$. The stiffness and support capacity are estimated using the equations proposed by Hoek et al (2002). The input parameters result in $p_s^{max}=1706$ kPa and $k_s=320.7$ MPa/m.

4.3 Reliability Analysis

The reliability analysis was performed analyzing each limit state function individually. In order to account for the effects of both limit state functions and to create a more realistic global failure criterion, in terms of probability of failure, the results of both curves were jointly considered by retaining the greater probability value of each individual function for each scenario analyzed.

The probability of failure versus lag distance of support installation (up to 15 m behind the face) was studied. The effect of the convergence limit on the probability of failure’s surface (LSF $g_2(X)$) and consequently in the global results was also addressed. The limit value for the radial linear strain convergence was set in 0.5%, 0.75%, and 1%. A value of 0.2% was also analyzed but no results were found since the support properties were not compatible with the equilibrium attainment with such a strict strain requirement. The results are shown below.

**Limit state function 1 ($g_1(X)$):**

The probability of failure of the support decreases, as expected, with the unsupported distance and becomes negligible for distances above 2 m, i.e. D/5 (Figure 4). This is a probability based illustration of the NATM principle that favours ground deformation in order to reduce the support load.

**Limit state function 2 ($g_2(X)$):**

Again accordingly with the NATM principles, the probability of excessive convergence evolves in the opposite sense from the probability of support failure. The importance of and the sensitivity to the strain limit criterion (Eq. 2) is illustrated in Figure 5.

One can appreciate the reliability achieved with a given limit value, one of the three close values of...
0.5%, 0.75% and 1%, considering different unsupported length values. As an example, let one admit that a minimum reliability index of 3 is required for safety reasons. For the assumed geotechnical conditions, one can observe that one should restrict the limit strain value to either 0.75% or 1%. Otherwise, the limit strain criteria (0.5%) would be violated too frequently, causing the implementation of corrective measures, e.g. shortening the unsupported length.

As for the joint consideration of both limit state function, the results below clearly suggest that the optimum design decision for a given pair \((\varepsilon_{\text{max}}, p_{\text{smax}})\) is attained at an intermediate distance lag. Although this optimum distance is dependent on the pair chosen, the lag distance is not quite sensitive to this choice (Figure 8). It would seem that, with the present scenario, it would be unsafe to construct the tunnel as the maximum value of the reliability index is hardly above 2.

Yet, a remark is in order about the meaning of the probability and reliability index values presented. Those are not to be understood as absolute values, rather as a sorting index for the situations in presence. One must remember that the present results correspond to a given choice of mechanical model (CCM), of constitutive models (M-C model), of statistical distributions (Gaussian) and a solution method (FORM). Also in a simplified way some aspects were considered deterministic rather than uncertainties, with proper distributions and statistical parameters.

5 APPlicability to FEM

In order to test the applicability of this methodology to finite element methods, a simple example was considered, regarding the same problem but considering only the LSF 1, evaluated for a lag distance of \(L = 2m\). The results of the analytical evaluation using CCM (see Figure 4) are listed in Table 2.

The response surface method (RSM) (Pula and Bauer, 2007) was used to build the LSF and the responses needed to calibrate this function were obtained using the commercial software PLAXIS v8.

<table>
<thead>
<tr>
<th>Function</th>
<th>(c') (kPa)</th>
<th>(\phi') (°)</th>
<th>(E) (MPa)</th>
<th>(\beta)</th>
<th>(p_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSF(_1)</td>
<td>1950.99</td>
<td>22.2</td>
<td>487.7</td>
<td>2.98</td>
<td>0.00146</td>
</tr>
</tbody>
</table>

The tunnel boundary was simulated using 60 line segments and the ‘Total Multiplier’ modulus was used to apply the required pressure on the tunnel walls. The finite element mesh is represented in Figure 8.

A total of 3 iterations with 27 calculations each were performed. The Eq. 4 shows the resulting limit state function and the Table 3 the design point and the corresponding reliability index (\(\beta\)) and probability of failure.

\[
g_1(X) = -0.129 - 5.4 \times 10^{-4} c' + 1.6 \times 10^{-7} c'^2 + 4.7579 \tan \phi' - 4.599 \tan \phi'^2 - 4.1 \times 10^{-4} E + 5.1 \times 10^{-8} E^2 - 9.5 \times 10^{-4} c' \tan \phi' + 1.8 \times 10^{-7} c' \cdot E + 7.4 \times 10^{-4} E \cdot \tan \phi' \tag{4}\]

<table>
<thead>
<tr>
<th>Function</th>
<th>(c') (kPa)</th>
<th>(\phi') (°)</th>
<th>(E) (MPa)</th>
<th>(\beta)</th>
<th>(p_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSF(_1)</td>
<td>1877.62</td>
<td>22.1</td>
<td>555.5</td>
<td>2.74</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Comparing the results from analytical and numerical analysis it can be concluded that both results are fairly close to each other not only regarding reliability index and probability of failure but also in the
way the design point vector deviates relatively to the midpoint.

The detailed explanation and the full set of results can be found in Faustino (2013).

6 CONCLUDING REMARKS

The effects of the distance from the support installation have been investigated regarding reliability analysis for two main issues, each one resulting in different limit state function.

The results show that different consequences result from the range of distances of the support installation and that the choice of which results in a lower probability of failure isn’t simple when analyzed simply a function at a time.

In this example, it is clear that a better understanding of the global effects can be obtained if both attained surfaces are analyzed together as each one evolves in a different way with the increasing of the distance.

For small distances, the support shapes the global surface as the ground pressure on the support remains high (and equal to its capacity). With the increase of this distance the importance of the displacements becomes higher, eventually shaping the global probability of failure’s surface.

If both the support and the displacements of the ground are addressed together it is also clear that a small gap between the tunnel face and the support position considerably reduces the risks of failure (collapse).

The numerical analysis proved to be a viable alternative, for this case, when used together with the Response Surface Methodology (RSM) approach.

Given the relatively simplicity of this analysis and the importance of the information that can be obtained, we believe that the answer to “RELIABILITY ANALYSIS OF TUNNEL CONVERGENCE: IS IT WORTH THE EFFORT?” is undoubtedly yes.

ACKNOWLEDGEMENT

The authors would like to thank the Portuguese Geotechnical Society for the sponsorship given to present this work.

REFERENCES


