Predictors of Stochastics Achievement in Primary School Pupils

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Abstract

We identify predictors for stochastic achievement for children aged 6 to 12 years (grades 1 to 4). More than 700 children from Austria, Germany, Switzerland and Hungary completed the KI(D)S-stochastics quiz. For primary school pupils (grades 1-4) neither social background nor sex had a significant influence on stochastics achievement. In contrast, grade and interest in mathematics were good predictors for stochastics achievement even when controlling for confounding variables. Grade (i.e., years of instruction in mathematics) was a stronger predictor of stochastic achievement than age. Ability in mathematics was correlated with performance in combinatorial tasks, and with performance in interpreting graphs. IQ was significantly correlated with the overall test score.

Keywords: Stochastics in primary school; stochastic competence; KI(D)S-quiz;

1. Introduction

Stochastics is a subfield of mathematics with important applications in everyday life. It deals with the description and investigation of random experiments and with random temporal and spatial events (e.g. betting, gambling, throwing a dice,…). Probability is the core concept of stochastics and an essential feature of everyday mathematical knowledge. Modern live is replete with decisions requiring some basic competence in stochastics.

Despite its importance, stochastics education is implemented only late in traditional curricula (Milcke & Martignon , 2007). While most high schools by now include stochastics and probability theory in their mathematics curricula, it is still debated how early and to what extent children should be taught stochastics in primary and lower secondary school. Most curricula in lower grades do not include concepts of stochastics, therefore, statistical education in primary and secondary schools is still rudimentary, even though a lot of questions in modern daily life of pupils are stochastical in nature (e.g., graphs, games, wheels of fortune, …). As a consequence stochastics items are difficult even for 15/16 year old students: In Austria and Germany, only about 30 % of students this age were able to solve PISA 2003 test items related to stochastics e.g. *"Consider two boxes, A and B. Box A contains three marbles, of which one is white and two are black. Box B contains seven marbles, of which two are white and five are black. You have to pick a marble from one of the boxes with your eyes covered. Which box should you choose if you want a white marble?" (*Bryant & Nunes). Laplace (1814) predicted that in such problems people would choose the box with the larger prop of white marbles based on "unaided common sense". However, results of the PISA 2003 survey demonstrate that this may not be the case. Indeed, solving probabilistic problems correctly in mathematical terms has previously been described as very difficult even for students, and even after instruction (Freudenthal, 1973; Hope et al., 1983). In addition, modern cognitive psychology has accumulated abundant empirical evidence that people tend to avoid basing their decisions on probability theory, but rather use a variety of heuristics (Gigenrenzer, 1999; 2000; 2002). Stochastics items differ from classical mathematical tasks, which mainly depend on the processing of number magnitude (de Smedt et al., 2009) and problem solving/reasoning abilities (Büchter, 2010). These classical mathematical abilities (e.g., addition, subtraction, division, multiplication, bigger-smaller tasks, or numerical series tasks) are more deductive in nature and mostly lack items on probability and combinatorics. However, distinct brain systems have been hypothesized to underlie deductive and probabilistic reasoning (Parsons & Osherson, 2001). Therefore knowledge on the relationship between mathematics and stochastics achievement is poor.

As we have said, much is known about achievement in mathematics, but hardly anything is known about school achievement in stochastics. The most prominent predictors of mathematics performance are mathematical interest (Doll & Prenzel, 2004; Wallner-Paschon, 2013), age (Doll & Prenzel, 2004), gender (Hyde, Fennema & Lamon, 1990; Filzmoser & Suchan, 2013; Wallner-Paschon, 2012; Schwantner & Schreiner, 2010) and social background (Schreiner, 2013; Schwantner & Schreiner, 2010).

In addition, in adults, mathematical knowledge positively impacts achievement in introductory statistics courses, which include a good amount of stochastics (Tremblay et al., 2000; Harlow et al., 2002; Chiesi & Primi, 2009; Pletzer et al., 2010). However, as we are not aware of any systematic research on the relationship between stochastic achievement and mathematical achievement during development, our goal is to provide such research.

Gender differences in mathematical achievement have been a matter of debate for decades. There is a longstanding tradition of studies describing a male superiority in mathematical tasks among adults (e.g. Benbow, 1988; Casey et al., 1997; Sappington & Topolski, 2005; Halpern et al., 2007). Gender differences in mathematics emerge early and have been observed throughout the course of mathematics education. According to Filzmoser and Suchan (2013), among 15/16 year old students, boys outperform girls in most of OECD-countries. Also among 4th graders (age 9/10 years), boys perform significantly better in mathematics than girls in most of OECD countries, and in EU average (Wallner-Paschon, 2012). Even among 3rd graders, boys do better than girls in mathematics (Penner & Paret, 2008) whereas in kindergarten sex differences were only present at the top 5% of the distribution. Hyde, Fennema and Lamon (1990) stress, that the lower performance of women in problem solving especially in high school requires attention. However, it has also been argued that these differences may be restricted to complex tasks or highly gifted students (Hyde, 2005; Hyde et al., 1990; Spelke, 2005).

In nearly all EU countries mathematics achievement is significantly correlated with social background. The higher the social background, the better is mathematic achievement for students aged 15/16 years and also for 4th graders (Schreiner, 2012; Schreiner, 2013). Social background explains between about 7% (Finland) and 20% (Luxemburg) of the variance in EU-countries in 15/16 year old students (Schreiner, 2013). In 4th grade, social background has also a significant impact on achievement. In general, educational surveys find that children with high socioeconomic status achieve significantly better in mathematics than children with low socioeconomic status (Schreiner, 2012).

To the best of our knowledge, no study so far has investigated the interrelation of mathematical skills and stochastics achievement at the beginning of school education. To address this issue we developed a quiz assessing stochastic achievement of children in grades 1 to 8 (Kipman, Kühberger & Fritz, 2014, in review). In the present study, the quiz was administered to a large sample of primary and secondary school pupils to investigate the impact of age, gender, social background, mathematical interest and mathematical ability on stochastics achievement.

2. Material & Methods

2.1. Participants

We recruited 704 primary school pupils (50% female; mean age: 8.52 ± 1.17 years, range: 6 – 12 years) for the main study. To evaluate the stability of our findings in later education we recruited 198 additional students from lower secondary school (43% female; mean age: 13.37 ± .70 years, range: 11 – 15 years; grades 7 and 8). All parents gave their informed written consent for their child's participation in the study.

2.2. Predictor Variables

Social background was assessed by pupil's judgements of the number of books in their home. This variable has been demonstrated to be a very good predictor of socioeconomic status (Rost & Wessel, 1994; Elley, 1994; Lietz, 1996; Schwippert, 2002; Bos et al., 2003). *Mathematical ability* and *mathematical interest* was rated by teachers on a scale from 1 to 5. We did not use grades and self-ratings of interest since grades in mathematics and student's self-rated enjoyment / interest of mathematics show only little variance at this young age and thus have only limited predictive power of actual mathematical abilities and mathematical interest. Due to the lack of student's familiarity with the concept of stochastics, interest in stochastics was not assessed. Grade was used as a proxy for previous *experience with mathematics*. Importantly, mathematical ability was rated in reference to students of the same grade. Teachers also rated *reading skills* of students on a scale of 1 to 5. IQ-data were collected from a subsample of 195 pupils aged 8 to 10 years using the HAWIK IV.

2.3. Stochastics abilities – The KI(D)S-Quiz

Stochastics achievement of students was assessed using the KI(D)S-Quiz, a reliable 46 item instrument developed by Kipman, Kühberger, and Fritz (2014). Cronbachs α with these items was α = .88 (Kipman, Kühberger & Fritz, 2014, in review). The quiz consists of five subtests: (i) terms, (ii) tables, (iii) graphs, (iv) random experiments and (v) combinatorics (Kipman, Kühberger & Fritz, 2014, in review), to be completed during a regular mathematics class in 45 minutes. Examples for each subtest are given below.

(i) Terms: assesses children's knowledge of the terms "possible", "certain", "impossible" and "probable" (e.g., "It is possible that a black marble is drawn" - if a box contains marbles of different colours).

(ii) Graphs: requires matching of a text to an appropriate graphical exhibit (e.g. „There are 15 boys and 8 girls in a classroom" – to be translated in a graphical form, and vice versa).

(iii) Tables: counting various activities of a fictional character in a story and answering questions like „Which vehicle has the boy seen most often?".

(iv) Random-experiment: deciding which outcome is more likely when presented with a random experiment (e.g. wheels of fortune, dices).

(v) Ccombinatorics: sampling with and without replacement and an „n over k" – task (e.g. „An ice-cream parlor sells 4 kinds of ice-cream: chocolate, vanilla, strawberry and nut. You want to buy 2 scoops and don`t want to buy one kind twice, how many possibilities do you have? "

3. Results

Linear regression analyses were performed to evaluate the impact of age, grade (i.e. mathematical experience), gender, social background, mathematical ability and mathematical interest on stochastic achievement (total score as well as subtest scores). The predictive strength of each variable was first assessed in a univariate model (e.g. impact of gender alone) and then in a multivariate model controlling for the other variables respectively (e.g. impact of gender controlling for age, grade, social background, mathematical abilities & mathematical interest). As understanding of the instructions may substantially limit performance in the test, reading skills were controlled for in all analyses.

In the following, we describe the influence of all predictors on the total test score and all subtest scores with and without controlling for confounding variables for primary (Table 1), and secondary school (Table 2). We also provide a path model of the direct and indirect influences of all variables on stochastics achievement for primary school pupils (see Figure 1).

A first important finding is that stochastics achievement increased with the number of years of schooling, since age and grade are significant univariate predictors of stochastics achievement (total score and all subtest scores, except Random Experiments). The association was strongest for graphs and combinatorics. Controlling for grade in a multivariate analysis eliminated the age effect.

The next important finding was that mathematical ability influenced stochastics achievement (total score) in primary school (total score and all subscores except graphs).

After controlling for confounding variables ability was no longer significant for the total score but remained significant in the terms and the tables subtests.

Mathematical interest was a significant predictor of stochastics achievement, even after controlling for confounding variables in primary school. The higher student's mathematical interest was the higher was their performance in the Kids-Quiz.

Overall, gender and social background did not affect stochastics achievement, even after controlling for age, grade, social background, ability in mathematics, interest in mathematics and reading ability.

If at all, a small positive impact of social background could be observed in the Random Experiments and Graphs subtest. Note, however, that in secondary school social background had significant impact on total score (but not on all subtest scores; Combinatorics and Random Experiments were not associated with social background).

IQ was significant correlated with stochastics achievement (r= .199, p < .01), in particular with the subtests Graphs (r= .169, p< .05), Tables (r= .145, p < .05), Random Experiments (r= .180, p < .05), and combinatorics (r= .207, p < .01).

To provide an overall pattern for primary school we combined the variables in a path model (see Figure 1). As can be seen, background variables like social background, gender or age did not have a reliable influence on interest in mathematics. Interest in mathematics and ability in mathematics are strongly correlated. Controlling for background variables, impact of interest in mathematics on stochastics achievement is rather small. Influence of ability in mathematics on stochastics achievement is - after controlling for interest and background variables – also rather small. Age is strongly correlated with stochastics achievement.

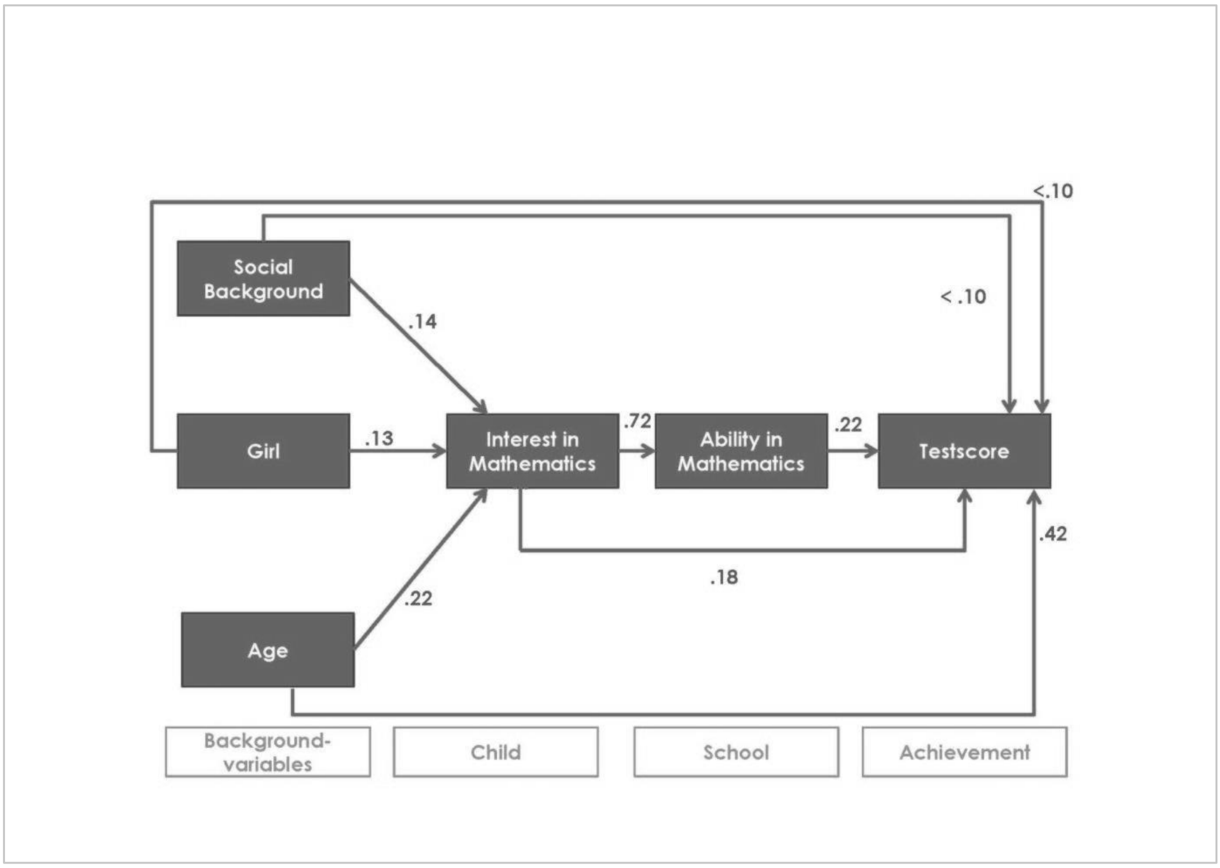

Figure 1: Path model for primary school

Discussion

The present study is the first to identify predictors of stochastics achievement in early education. We evaluated the impact of, gender, background (gender, social background), schooling (age, grade), idiosyncratic variables related to formal thinking (mathematical interest, mathematical ability) on stochastics achievement as measured by the KI(D)S-quiz.

In primary school, grade was a stronger predictor than age, indicating that schooling had a more profound effect on stochastics achievement than maturation in general or out-of-school experience. This indicates that even the better educated parents do not promote their primary school children in stochastics. As children grow older, children of higher social background may gain more experience with educational games and lotteries, allowing them to better judge probabilities. It seems as if the importance of probabilistic reasoning is strikingly underestimated, not only in school curricula, but in society as a whole. Furthermore, it could be that most parents had little experience with statistics and stochastics education themselves, even less than their children will have, once they enter high school. As their own probabilistic judgements are widely based on heuristics, they do not educate their children in stochastic thinking.

Importantly, while mathematical interest is a strong predictor of stochastics achievement, mathematical ability did only affect combinatorics, but not any of the other subtests. This finding appears to be in contrast with findings in adults, showing a strong influence of mathematical knowledge on achievement in introductory statistics. However, mathematical knowledge in adults probably reflects mathematics experience collected during the schooling years, which may vary tremendously. Consequently, even though students who are interested in mathematical problems, may also be interested in probabilistic problems, the deductive reasoning skills allowing them to excel in mathematics do not influence their stochastics achievement. This finding is in line with brain imaging data showing that deductive and probabilistic reasoning rely on different brain circuits.

In summary, stochastic ability in the early school years is an important, yet a largely unknown topic. Here we show that, although related to standard mathematics, stochastic achievement but that it goes well beyond mathematics. Our findings imply that a focus on deductive reasoning is not enough to provide students with the necessary skills to adequately deal with stochastic and statistics problems in high school or college. Therefore, it is of uttermost importance to include stochastics in primary school curricula and allow students to become as familiar with the concepts of stachastics over the years of their schooling experience as they become familiar with basic arithmetics.

Table 1: Regressions (primary school)

	β	β ‘	R^2	F
Total Score				
Gender	.078	.027	10.5%	33.06
Social background	.016	.013	9.3%	28.39
Age	.406***	.072	26.0%	95.94
Grade	.513***	.587***	36.2%	156.46
Interest in mathematics	.214***	.214***	13.5%	43.77
Ability in mathematics	.204***	.053	12.4%	39.59
Terms				
Gender	.045	.016	4.2%	12.85
Social background	.025	.045	3.3%	10.15
Age	.297***	.089	12.7%	40.23
Grade	.379***	.173**	18.4%	62.48
Interest in mathematics	.159**	.016	5.9%	18.11
Ability in mathematics	.133*	.105*	5.0%	15.42
Graphs				
Gender	.096*	.049	4.7%	14.5
Social background	.092*	.087*	4.8%	14.41
Age	.521***	.212**	30.2%	118.26
Grade	.529***	.370***	31.9%	128.88
Interest in mathematics	.056	.160**	4.0%	12.45
Ability in mathematics	.017	.070	3.8%	11.78
Tables				
Gender	.018	.004	6.4%	19.63
Social background	.062	.016	6.5%	19.77
Age	.221***	.267***	11.1%	34.63
Grade	.352***	.594***	18.8%	64.26
Interest in mathematics	.137**	.039	7.8%	24.12
Ability in mathematics	.193***	.133*	8.6%	26.67

Random experiments				
Gender	.055	.001	2.4%	7.86
Social background	.119**	.103*	3.3%	10.25
Age	.002	.197	2.2%	7.13
Grade	.072	.168*	2.7%	8.48
Interest in mathematics	.313***	.224***	9.6%	30.20
Ability in mathematics	.296***	.118	3.4%	22.75
Combinatorics				
Gender	.063	.035	9.4%	29.26
Social background	.040	.015	8.6%	26.39
Age	.309***	.012	18.8%	63.54
Grade	.385***	.397***	23.9	86.72
Interest in mathematics	.129**	.174**	10.2%	32.21
Ability in mathematics	.106*	.012	9.6%	30.15

Table 2: Regressions (lower secondary)

	β	β ‘	R^2	F
Total Score				
Gender	.335***	.378***	22.5%	10.46
Social background	.345**	.235*	22.6%	10.5
Age	.357**	.439**	23.7%	11.11
Grade	.264*	.119	17.8%	8.05
Interest in mathematics	.322***	.178	21.2%	9.76
Ability in mathematics	.035	.068	11.1%	5.07
Terms				
Gender	.281*	.320**	5.8%	3.01
Social background	.478***	.422***	20.1%	9.15
Age	.210	.396*	2.1%	1.71
Grade	.069	.279	1.3%	0.43
Interest in mathematics	.281*	.070	5.5%	2.9
Ability in mathematics	.164	.105	0.5%	1.17
Graphs				
Gender	.104	.162	17.2%	7.74
Social background	.240*	.126	21.7%	9.99
Age	.309**	.219	25.6%	12.17
Grade	.239*	.049	21.7%	9.99
Interest in mathematics	.324**	.206	26.4%	12.67
Ability in mathematics	.107	.070	17.2%	7.77
Tables				
Gender	.301**	.270*	22.1%	10.20
Social background	.340**	.270*	24.0%	11.28
Age	.262*	.265	19.6%	8.92
Grade	.284*	.014	20.6%	9.46
Interest in mathematics	.199	.138	16.7%	7.50
Ability in mathematics	.242*	.262*	18.8%	8.51
Random experiments				

Gender	.398**	.491***	23.5%	11.01
Social background	.044	.155	7.4%	3.62
Age	.293*	.301	15.8%	7.12
Grade	.258*	.062	13.8%	6.19
Interest in mathematics	.069	.002	7.7%	3.72
Ability in mathematics	.089	.023	8.1%	3.86
Combinatorics				
Gender	.093	.073	0.6%	1.19
Social background	.034	.025	2.9%	0.94
Age	.244	.322	5.6%	2.94
Grade	.170	.163	2.5%	1.84
Interest in mathematics	.258*	.288*	6.3%	3.18
Ability in mathematics	.237	.349*	5.5%	2.88

References

Benbow, C. (1988). Sex differences in mathematical reasoning ability in intellectually talented preadolescents: their nature, effects, and possible causes. *Behavorial and Brain Sciences,* (11), 169–183.

Bos, W., Lankes, E., Prenzel, M., Schwippert, K., Walther, G., & Valtin, R. (2003). *Erste Ergebnisse aus IGLU.: Schülerleistungen am Ende der vierten Jahrgangsstufe im internationalen Vergleich.* Münster: Waxmann.

Bryant, P. & Nunes, T. *Children's understanding of probability*. Retrieved from http://www.nuffieldfoundation.org/sites/default/files/files/Nuffield_CuP_FULL_REPORTv_FINAL.pdf (08.01.2014)

Casey, M., Nuttall, R., & Pezaris, E. (1997). Mediators of gender differences in mathematics college entrance test scores: A comparison of spatial skills with internalized beliefs and anxieties. *Developmental psychology,* (33), 669–680.

Elley, W. (1994). *The IEA Study of Reading Literacy. Achievement and instruction in thirty-two school systems.* Exeter: Pergamon.

Filzmoser, S., & Suchan, B. (2013). Mathematik: Unterschiede zwischen Mädchen und Burschen. In U. Schwantner, B. Toferer, & C. Schreiner (Eds.), *PISA 2012. Internationaler Vergleich von Schülerleistungen.* (pp. 24–25). Graz: Leykam.

Gigerenzer, G. (1999). Mentale Fakultäten, methodische Rituale und andere Stolpersteine. *Zeitschrift für Psychologie*, 287–297.

Gigerenzer, G. (2000). *Adaptive thinking. Rationality in the real world.* Oxford: Oxford University Press.

Gigerenzer, G. (2002). *Das Einmaleins der Skepsis.: Über den richtigen Umgang mit Zahlen und Risiken.* Berlin: Berlinverlag.

Halpern, D., Benbow, C., Geary, D., Gur, R., Hyde, J., & Gernsbacher, M. (2007). The science of sex differences in science and mathematics. *Psychological Science in the Public Interest.*, (8), 1–51.

Hyde, S., Fennema, E., & Lamon, S. (1990). Gender differences in mathematics performance: A meta analysis. *Psychological Bulletin*, (107), 139–155.

Lietz, P. (1996). *Changes in reading comprehension across cultures and over time.* Münster: Waxmann.

Penner, A., & Paret, M. (2008). Gender Differences in Mathematics Achievement: Exploring the Early Grades and the Extremes. *Social Science Research*, (37), 239–253.

Sappington, J., & Topolski, R. (2005). Math performance as a function of sex, laterality, and age of pubertal onset. *Laterality: Asymmetries of Body, brain, and Cognition*, (10), 369–379.

Schreiner, C. (2012). Bildung der Eltern und Schülerleistungen in Österreich. In B. Suchan, C. Wallner-Paschon, S. Bergmüller, & C. Schreiner (Eds.), *PIRLS & TIMSS 2011. Schülerleistungen in Lesen, Mathematik und Naturwissenschaft in der Grundschule.* (pp. 48–49). Graz: Leykam.

Schreiner, C. (2013). Familiärer Hintergrund und Leistung. In U. Schwantner, B. Toferer, & C. Schreiner (Eds.), *PISA 2012. Internationaler Vergleich von Schülerleistungen* (pp. 46–47). Graz: Leykam.

Schwantner, U., & Schreiner, C. (Eds.). (2010). *PISA 2009.: Internationaler Vergleich von Schülerleistungen.* Graz: Leykam.

Schwantner, U., Toferer, B., & Schreiner, C. (Eds.). (2013). *PISA 2012.: Internationaler Vergleich von Schülerleistungen*. Graz: Leykam.

Schwantner, U. (2010). *PISA 2009 - Internationaler Vergleich von Schülerleistungen: Erste Ergebnisse Lesen, Mathematik, Naturwissenschaften*. Graz: Leykam.

Spelke, E. Sex differences in intrinsic aptitude for mathematics and science? *American Psychologist*, *2005*(60), 950–958.

Suchan, B., Wallner-Paschon, C., Bergmüller, S., & Schreiner, C. (Eds.). (2012). *PIRLS & TIMSS 2011.: Schülerleistungen in lesen, Mathematik und Naturwissenschaft in der Grundschule.* Graz: Leykam.

Suchan, B., Wallner-Paschon, C., Bergmüller, S., & Schreiner, C. (Eds.). (2012). *PIRLS & TIMSS 2011.: Schülerleistungen in Lesen, Mathematik und Naturwissenschaft in der Grundschule.* Graz: Leykam.

Wallner-Paschon, C. (2012). Mathematik: Leistungsunterschiede zwischen Mädchen und Buben. In B. Suchan, C. Wallner-Paschon, S. Bergmüller, & C. Schreiner (Eds.), *PIRLS & TIMSS 2011. Schülerleistungen in Lesen, Mathematik und Naturwissenschaft in der Grundschule.* (pp. 30–31). Graz: Leykam.