Understanding Labour Market Frictions: An Asset Pricing Approach

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Abstract

Labour market friction is viewed in terms of the market value of an employed worker as opposed to the position of the Beveridge curve. This market value of an installed worker, which I call Tobin’s $Q$ of a worker, is inversely proportional to the average quality of the match between employers and workers. Based on this measure, I find that the labour market friction rises during a period of productivity boom. This phenomenon is indirectly supported by the data where it is found that the relative value of a worker with respect to tangible capital shows a positive association with the TFP. The model suggests that firms may be compromising the quality of a skill match during a period of tight labour market condition.

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1 Without implicating I would like to thank John Cochrane for inspiring me to undertake this project. Thanks are due to Martin Robson and Weshah Razzak for constructive comments. Anurag Banerjee is acknowledged for help on a technical issue. The paper significantly benefited from insightful comments from an anonymous referee. Thanks are also due to Mauricio Armellini and Soyeon Lee for able research assistance.
1. Introduction

The relative price of investment to consumer goods has significantly declined over time in the US. This decline is particularly noticeable in the 80s, which coincided with the great period of moderation of output volatility. Figure 1 plots the ratio of US producer price index of finished capital goods to the consumer price index. Following the oil shock in the early 70s, there is a steady decline in this relative price of investment goods, which reconfirms the decrease in capital market frictions in the 80s. A number of papers ascribe this recent decline to elimination of investment frictions (Greenwich, Hercowitz and Krusell, 2000; Chari, Kehoe and McGrattan, 2007). Although there is a near consensus that the degree of capital market frictions in the US has substantially decreased recently, less is known about labour market frictions.

Following the work of Pissarides (1985), by labour market friction I mean the degree of mismatch between the worker and the employer. Little is known about this job-matching variable at the aggregate level. A sizable literature focuses on the behaviour of the unemployment-vacancy relationship (known as the Beveridge curve) as a measure of this friction. There are both empirical and theoretical limitations of this Beveridge curve based approach. Vacancies are usually measured by the help-wanted index which is less reliable particularly after the internet revolution when job openings are mostly available online. Valletta (2005) attempts to correct this deficiency by creating a synthetic job vacancy ratio and argues that the Beveridge curve has shifted inward in the 80s after an outward shift in the 70s. Shimer (2005) argues that the vacancy-unemployment ratio has a remarkable volatility which makes it difficult to arrive at a definitive conclusion about the time path of the labour market frictions.²

Due to this limitation of the Beveridge curve based analysis I suggest a new measure of labour market frictions based on asset pricing principles. From the firm’s perspective, a higher probability of a worker-firm match can be thought of as higher productivity of the firm’s search effort. If firms can efficiently search such that the likelihood of a worker-firm match is higher, it will lower the market value of the already employed workers. The reason is simply that the incumbent workers are easily

² Hornstein et al. (2005) extend Shimer’s (2005) work and find additional problems in replicating the observed unemployment-vacancy fluctuations using the extant matching models.
replaceable because of the higher productivity of the firm’s search effort. Therefore, these currently employed workers receive less rent to their skills in an environment where the skill-match probability is higher. Thus one expects that the skill premium of an incumbent worker will be lower when there is less labour market friction.

The market value of an incumbent worker can be thought of as the $Q$ value, which is reminiscent of Tobin’s (1969) $Q$ measure. While Tobin’s $Q$ measure is mostly applied to physical or tangible capital, I propose to apply this measure to human capital and use it to derive a new measure of labour market friction. A higher $Q$ of an incumbent worker thus signals a higher friction in the labour market. This asset-based measure of labour market friction is motivated by Chari, Kehoe and McGrattan (2007) (C-K-M hereafter) who suggest a measure of investment friction based on relative price of investment goods. The scope of my paper differs in two important ways from C-K-M (2007). First, while the principal focus of C-K-M is business cycle accounting in terms of various frictions, my scope is limited to the understanding of labour market friction alone. Such a friction is interpreted as a wedge on the human capital investment of the firm which is driven by a fundamental TFP shock. Second, C-K-M does not view labour employment in terms of an investment decision of the firm.  

I employ a production based asset-pricing model based on the work of Merz and Yashiv (2007) and Cochrane (1991) to derive the Tobin’s $Q$ of an incumbent worker. By construction, this Tobin’s $Q$ is inversely related to the firm’s recruitment productivity. The $Q$ of the worker shows endogenous fluctuations driven by the TFP shock. Parallel to investment friction, in my model, more friction in the labour market means a higher Tobin’s $Q$ of the existing worker. Using a calibrated version of this model, I estimate the economy-wide matching probability based on the US aggregate data and find that it is strongly countercyclical. This basically means that the quality of the worker-employer match deteriorates during a boom. The implication is that firms compromise on the match quality in hiring new employees in an economy with a tight labour market.

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3 C-K-M (2007) provide a framework to account for business cycles in terms four frictions, namely efficiency wedge, labour wedge, investment wedge and the government consumption wedge. Their methodology of business cycle accounting has been recently criticised by Christiano and Davis (2006) referred C-D hereafter. While both C-K-M and C-D papers are very insightful in business cycle accounting and are fairly general in their scope, neither C-K-M nor C-D have human capital investment in their models and neither of them address the issue of labour market friction in a production based asset pricing framework as I do.
The plan of the paper is as follows. In the following section, I report some stylized facts about the time series behaviour of the relative price of labour in terms of capital. In section 3, a production-based asset-pricing model is laid out to show the precise relationship between the labour market friction and the value of a worker. Section 4 reports some calibration results. Section 5 concludes.

2. Capital and Labour Market Frictions: Some Stylized Facts
Motivated by the relative price-based measure of input frictions as in Greenwood et al. (2000), I calculate the relative price of a worker with respect to capital for the US economy over the period 1948-2001 to arrive at a measure of labour market friction relative to capital market friction. This relative price is measured by the ratio of the annual index of compensation per worker to the producer price index of finished capital goods over the period 1948-2001 taking 1992 as the base year. Data for compensation per worker came from Hall (2005) who compiled these data from the Bureau of Labour Statistics (BLS). The producer price index of finished capital goods came from the US Department of Labour, BLS. Figure 2 plots the series. The relative price of a worker shows a steady increase except for the period of the oil shocks during 1973-74 when all producer prices increased.

In the next step, I examine the cyclical behaviour of the relative price of a worker. I use the total factor productivity (TFP) as an indicator of the business cycle. In order to verify the robustness of the key results, two series for TFP are used. The first series is the Solow residual based on non-farm output, aggregate hours worked and non-residential fixed assets covering the period 1960-2001 for which an overlapping series for all three are available. 4 The second slightly longer series is the annual manufacturing multifactor productivity index obtained from the Bureau of Labour Statistics. This series is used as a proxy for TFP to check for the robustness of results. The correlation coefficient between these two TFP series is 0.95. Figures 3(a) and 3(b) plot the total factor productivity (TFP) index and the relative price of worker after taking out a linear trend component from each series. Both plots show very similar pattern. The cyclical component of the

4 The Solow residual is constructed by running a regression of the log of real GDP in the non-farm business sector, (series PRS85006043 from BLS), aggregate hours worked, as above, and the real non-residential fixed assets obtained from BEA Survey of Current Business.
value of worker positively correlates with the cyclical component of the TFP shock. The correlation coefficient between these two series is 0.73 for Figure 3(a) and 0.66 for Figure 3(b). The relative price of worker rises during a period of TFP boom.5

In the rest of the paper, I argue that this relative price of worker with respect to capital can be interpreted as the Tobin’s $Q$ of an incumbent worker. I also argue that this procyclical behaviour of a worker’s Tobin’s $Q$ is driven by a decrease in the quality of the match between workers and the employers during an expansion. This quality of the match is measured by the productivity of the recruitment efforts. As the labour market tightens during a boom, firms start compromising on the quality of the match while recruiting. This makes already employed workers more valuable to the firm. Based on this analysis, I argue that the Tobin’s $Q$ of a worker is a reasonable measure of labour market friction as opposed to unemployment-vacancy ratio. To make this point transparent, in the next section, I focus on the production sector of the economy and develop a labour based asset-pricing model.

3. The Model

The production-based asset-pricing model is an adaptation of Merz and Yashiv (2007).6 The production sector consists of identical firms sharing the same production and investment technology facing a market wage rate, $w_t$. The timeline is as follows. At the start of date $t$, the firm observes a TFP shock $\varepsilon_t$ and produces output with the predetermined tangible capital $K_t$ and the human resources $N_t$ using the following Cobb-Douglas production function:

$$Y_t = \varepsilon_t K_t^\alpha N_t^{1-\alpha} \quad (1)$$

where $\alpha$ is the capital share in output. The firm then disburses the existing employees a real wage of $w_t$. Finally it undertakes two types of investment decisions: investment in

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5 The procyclical movement of the value of worker is robust to the choice of detrending method. I also looked at the correlation between Hodrick-Prescott detrended series for real GDP and the value of worker. The correlation coefficient between these two series is 0.50.

6 Merz and Yashiv (2006) use a production based asset-pricing model of the type pioneered by Cochrane (1991). Their innovation is to show that the market value of a firm can be decomposed into the value of capital and the value of labour.
tangible capital $I_t$ and posting of new vacancy, $V_t$. The cost of posting new vacancies, $X_t$ is proportional to the number of posting as follows:$^7$

$$X_t = aV_t; \quad \text{with } a > 0$$  \hspace{1cm} (2)

Investment in tangible capital augments firm’s physical capital following a standard linear depreciation rule:

$$K_{t+1} = (1 - \delta)K_t + I_t$$  \hspace{1cm} (3)

where $\delta$ is the constant rate of depreciation of physical capital.

Regarding the latter investment, I follow Merz and Yashiv (2007), to postulate the following law of motion for the employees:

$$N_{t+1} = (1 - \psi)N_t + q_tV_t$$  \hspace{1cm} (4)

where $\psi \in (0,1)$ is an exogenous job destruction rate, and $q_t$ is the probability that a posted vacancy will be filled or equivalently it is the match probability between a worker and an employer. Alternatively $q_t$ can also be interpreted as the quality of the match because it is positively related to the productivity of a firm’s spending on recruitment.$^8$ It will be shown later that $q_t$ is endogenous in this model and determined by the firm’s valuation of a worker, which in turn depends on economic fundamentals. Throughout my analysis I ignore any convex adjustment costs of changing capital and labour. Hall (2004) finds that adjustment costs are relatively minor and do not explain large part of the variation of the corporation value.

The representative firm facing a constant discount factor $\rho$ solves the following problem$^9$:

$$\text{Max } E_0 \left[ \sum_{t=0}^{\infty} \rho^t \{ \varepsilon_t K_t \alpha N_t^{1-\alpha} - w_t N_t - X_t - I_t \} \right]$$  \hspace{1cm} (P)

s.t. \hspace{1cm} (2) through (4), given $K_0, N_0$.

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$^7$ An introduction of a quadratic posting cost will give rise to nonlinearity which is akin to employment adjustment cost. In principle, one does not expect the main results about Tobin’s $Q$ of a worker to change much except that the firm needs to worry about a secondary cost of adjusting employment while making recruitment.

$^8$ Note that the marginal return to recruitment spending is: $\frac{\partial N_{t+1}}{\partial X_t} = q_t / a$.

$^9$ I ignore any convex adjustment cost in this benchmark model. There is, however, some built in adjustment cost of shifting resources from tangible to intangible capital. The firm incurs a relative price of $1/q_t$ to switch from tangible to intangible investment.
The TFP shock \( \varepsilon_t \) is specified as a geometric random walk as follows:\(^{10}\)

\[
\ln \varepsilon_{t+1} = \ln \varepsilon_t + \xi_{t+1}
\]  
\[ (5) \]

where \( \xi_{t+1} \sim N(0, \sigma^2) \)

The first order conditions with respect to \( I \) and \( X \) are as follows:

\( I: \)  
\[
1 = \rho E_t \left[ \varepsilon_{t+1} \alpha k_{t+1}^{-\alpha-1} + 1 - \delta \right]
\]  
\[ (6) \]

\( X: \)  
\[
aq_{t-1} = \rho E_t \left[ \varepsilon_{t+1} (1 - \alpha) k_{t+1}^{-\alpha} - w_{t+1} + (1 - \psi) aq_{t+1}^{-1} \right]
\]  
\[ (7) \]

where \( k_t \) is the capital/employment ratio at date \( t \). Given the random walk nature of the TFP shock, it is straightforward to verify that the capital-employment ratio is:

\[
k_{t+1} = \left[ \frac{\alpha \mu_1 \varepsilon_t}{1 - \rho(1 - \delta)} \right]^{1-\alpha}
\]  
\[ (8) \]

where

\[
\mu_1 = \exp \left( \frac{\sigma^2}{2} \right)
\]  
\[ (9) \]

The first order conditions (6) and (7) can be rewritten in the following valuation equation form:

\( I: \)  
\[
K_{t+1} = \rho E_t \left[ CF_{t+1}^k + K_{t+2} \right]
\]  
\[ (10) \]

\( X: \)  
\[
\frac{aN_{t+1}}{q_t} = \rho \left[ CF_{t+1}^n + aN_{t+2} \frac{q_{t+2}}{q_{t+1}} \right]
\]  
\[ (11) \]

where \( CF_t^k = \varepsilon_t \alpha k_t^{-\alpha-1} K_t - I_t \) and \( CF_t^n = \varepsilon_t (1 - \alpha) k_t^{-\alpha} N_t - w_t N_t - X_t \).

\(^{10}\) According to Prescott (1986) US TFP is a near random walk process while I assume that it is an exact random walk. Banerjee (2001) show that the first order forecast sensitivity due to difference stationary specification when the process is truly trend stationary is zero. See also Banerjee and Basu (2001) for a related paper. Moreover, I also performed a unit root test for the logarithm of the TFP series used in the following section. One cannot reject the null of a unit root.
Using (10) and (11) one can have the following value decomposition for the firm’s market value ($MV_t^M$)

$$MV_t^M = MV_t^K + MV_t^N$$  \hspace{1cm} (12)

where

$$MV_t^K = K_{t+1}$$  \hspace{1cm} (13)

$$MV_t^N = \frac{aN_{t+1}^t}{q_t}$$  \hspace{1cm} (14)

The Tobin’s $Q$ of capital ($MV_t^K / K_{t+1}$) is unity while the Tobin’s $Q$ of a worker ($MV_t^N / N_{t+1}$) is inversely proportional to the match probability $q_t$. The match probability $q_t$ drives a wedge between the Tobin’s $Q$ of capital and the Tobin’s $Q$ of labour.

Using (7) and (8), one can write the following Tobin’s $Q$ equation for a worker:

$$aq_t^{-1} = \rho(1-\alpha)\mu_t^{-1} \frac{1}{1-\alpha} \left[ \frac{\alpha\rho}{1-\rho(1-\delta)} \right]^{1-\alpha} - \rho E_t w_{t+1} + a \rho(1-\psi) E_t q_{t+1}^{-1}$$  \hspace{1cm} (15)

This valuation equation is just like a standard asset pricing equation. The worker is valued as an asset to the firm. The Tobin’s $Q$ of an installed (employed) worker is typically the expected present value of cash flows or surplus arising from his/her continued employment. This cash flow is the difference between worker’s expected productivity and the expected real wage.

**Wage Determination**

The wage determination story is the same as in Merz and Yashiv (2007). Real wage is determined by a Nash bargaining between worker and the firm assuming that both have equal bargaining power. Hiring an additional worker generates a surplus of for the firm $\{ MPN_t + (1-\psi)(a/q_t) - w_t \}$ and $(w_t - b_t)$ for the worker where $b_t$ is the unemployment
benefit which is the value of the outside option of the worker. In other words the real wage (w_t) is given by:

\[
w_t = \arg \max [MPN_t + (1 - \psi)(a / q_t) - w_t]^{1 - \phi} [w_t - b_t]^{1 - \phi}
\]  

(16)

where MPN_t is the marginal product of labour at date t and \( \phi \) is the bargaining strength of the firm vis-à-vis worker. Assuming that the unemployment benefit is an exogenously specified constant fraction of \( w_t \), wage equation in (16) is given by:

\[
w_t = (1 - \phi)[(1 - \alpha)e_k^\alpha + (1 - \psi)(a / q_t)]
\]  

(17)

**Vacancies and Tobin’s Q of a Worker**

Without any loss of generality, assume that a fixed fraction of the labour force participates in the labour market. In the absence of population growth this means that labour is inelastically supplied which I normalize at the unit level.\(^{11}\) Using (4) one gets the following vacancy equation

\[
V_t = \frac{\psi}{q_t}
\]  

(18)

Next substituting the wage equation (17) into (15) and using the method of undetermined coefficient, one arrives at the following solution for the worker’s Tobin’s Q:\(^{12}\)

\[
aq_t^{-1} = \Omega e_t^{1/(1-\alpha)}
\]  

(19)

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\(^{11}\) The assumption of an inelastic labour supply can be easily micro founded. Think of liquidity constrained workers who receive utility from consumption (c_t) and leisure \( T - l_t \) where \( l_t \) is the choice of work hours and \( T \) is a fixed time endowment. Each worker thus solves a static utility maximization problem: Max

\[
\ln c_t + \ln(T - l_t) \quad \text{s.t. } c_t = w_l l_t
\]

The optimal labour supply and leisure are \( T / 2 \). Given a fixed number of workers, the immediate implication is that the labour supply is a constant.

\(^{12}\) For the existence of the Tobin’s Q equation one requires a convergence condition that \( \mu^{1/(1-\alpha)} < 1 \) which is met for a sufficiently small value of \( \sigma^2 \). The conventional estimate of the variance of Solow residual is small. Prescott (1986) fixes it at .00763 which I use in the calibration section 4.
where \( \Omega = \left[ \frac{\alpha \rho (1 - \alpha) \mu_1^{1/(1-\alpha)}}{1 - \frac{0.5 \rho (1 - \psi) \mu_1^{1/(1-\alpha)}}{1 - \rho (1 - \delta)}} \right]^{\alpha/(1-\alpha)} \) \hspace{1cm} (20)

The appendix outlines the derivation of (19). Finally, plugging (19) into (18) one gets the following equilibrium vacancy equation:

\[ V_t = \frac{\psi \Omega}{a} \epsilon_t^{1/(1-\alpha)} \hspace{1cm} (21) \]

Two observations are in order. First, both Tobin’s \( Q \) of a worker in (19) and the equilibrium vacancy are positively related to the TFP. The intuition for this result goes as follows. A positive TFP shock at date \( t \) triggers an increase in the capital-employment ratio \( (k_{t+1}) \) in the following period (see equation 8). Due to the constant returns to scale property of the production function, a higher \( k_{t+1} \) lowers the marginal product of capital at date \( t+1 \), and raises the marginal product of the incumbent worker. Thus a higher TFP realization today basically signals a higher prospective relative return to human capital with respect to physical capital. In response to this, firms switch gear from physical investment to human capital investment, which means posting more vacancy (higher \( V_t \)). This increased demand for workers raises the value of the worker meaning lower search productivity \( q_t \). Basically firms compromise on the quality of the match during a period of a productivity boom.

Second, the model sheds some light about the volatility of vacancies relative to labour productivity as observed by Hornstein et al. (2005). The present model has no explicit labour productivity component; the only productivity variable is TFP, \( \epsilon_t \). The elasticity of vacancy with respect to TFP is \( 1/(1-\alpha) \) which exceeds unity. Vacancies are more volatile than TFP.\textsuperscript{13}

\textsuperscript{13} Although my model makes some progress in explaining why vacancies could be more volatile than TFP, the relative volatility of vacancy and productivity still remains a puzzle. Hornstein et al. (2005) point out that vacancies are about 20 times more volatile than TFP. To replicate this relative volatility, one requires an implausibly large value of the capital share parameter \( \alpha \).
4. Calibration

Parameter Values

There are six parameters of interest: $\alpha$, $\delta$, $\rho$, $\psi$, $\sigma^2$, $a$ and $\phi$. Following Prescott (1986), I set the benchmark values, $\alpha = .36$, and $\delta = 0.1$ (annual data), $\rho = .96$ and $\sigma^2$ is fixed at .00763. There is no published estimate of the parameter $\psi$. The closest one is the average job separation rate of 3% in the US economy over the period 1948-2001 found in Hall (2001). The remaining job posting cost parameter $a$ in equation (2) is scaled to ensure that the maximum value of job match probability $q_t$ equals unity. Without any loss of generality, the value of $\phi$ is fixed at 0.5.  

Model and Actual Tobin’s $Q$ of a Worker

Using the baseline parameter values and the observed series for the TFP, I next compute the model Tobin’s $Q$ of a worker. Since the model has no implication for growth, I detrend the TFP by passing a linear trend through it and then computing the residual. This TFP residual is then plugged into the Tobin’s $Q$ equation (19) given the baseline parameter values. This series is then compared with the detrended actual relative price of a worker. Figures 4(a) and 4(b) plot the results for both TFP series. To make these series comparable, all are scaled such that they are unity for the base year 1992. The model performs reasonably well in tracking down the business cycle fluctuations in the relative price of a worker. The correlation is 0.73 for Figure 4(a) and 0.67 in Figure 4(b). The relationship between actual and model’s $Q$ in Figures 4(a) and 4(b) are remarkably similar to the relationship between TFP and the actual Tobin’s $Q$ in Figures 3(a) and 3(b). This is expected because the TFP drives the model’s Tobin’s $Q$ as shown in equation (19).

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14 Changing the bargaining parameter has insignificant quantitative effect on the Tobin’s Q of a worker.

15 I also performed sensitivity analysis around the baseline parameter values which I do not report here for brevity. The correlation between the model and actual Tobin’s Q of a worker is reasonably robust to changes in the parameters.
An Estimate of the Employer-Worker Match Probability

In this section, I estimate the match probability $q_t$ based on the reduced form equation (19). Figures 5(a) and 5(b) plot this matching probability and the detrended TFP series. The matching probability declined during the 70s and then it revived in the 80s while TFP shows the opposing pattern. The matching probability increased during the 80s when there was a productivity slowdown. The result is robust with respect to the choice of the TFP series. These results reinforce my hypothesis that the quality of the match shows a countercyclical pattern.

Does Skill Match Worsen During a Boom?

The model predicts that the skill match and the consequent labour market friction worsen during a boom. There are three ways one can validate this prediction. First, the model predicts a procyclical movement of the Tobin’s $Q$ of a worker which holds up with the data as seen in Figure 4. The inverse of this Tobin’s $Q$ of a worker is proportional to the skill match probability. Second, my results are consistent with Valletta (2005) who finds that the US Beveridge curve shifted out during the 70s and then shifted back in during the 80s. The shift of the Beveridge curve appears countercyclical to productivity which is consistent with the model’s prediction. Third, there is also some international evidence of the procyclicality of the labour market friction. Hall and Scobie (2005) and Razzak (2007) document that the growth rates of output and labour productivity are relatively higher in Australia compared to New Zealand particularly after 1995. This productivity gain coincided with a period of higher capital intensity and a higher relative price of labour in Australia. Razzak (2007) confirms that during the same period Australia experienced an increase in the number of vacancies as well. These international stylized facts accord well with my model’s prediction that the labour market friction increases during a productivity boom.

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16 The TFP series is also normalized at unity taking 1992 as the base year.
17 A lesser productive match during a tighter labour market means that the firm strikes a quantity-quality trade off which could have an adverse effect on TFP. While this is a possibility, I do not model TFP in here. In my model, the TFP is the fundamental shock which means that the causality runs from TFP to the labour market friction not the other way round. In a future paper, one can investigate how this quantity-quality trade off could impact TFP.
**Investment Wedge vs. Capital Wedge**

In the present setting, the labour market friction is viewed as the market value of an incumbent worker. The skill match probability \( q_t \) drives a wedge between the Tobin’s \( Q \) of human capital and the Tobin’s \( Q \) of physical capital. In other words, the labour market friction is nothing but a wedge on the firm’s investment in human capital vis-à-vis physical capital. This wedge-view of labour market friction draws on the business cycle accounting (BCA) methodology as in Chari, Kehoe and McGrattan (2007). Christiano and Davis (2006) point out a major flaw of this BCA principle. Since investment has an intertemporal dimension, any wedge on it depends on future expectations of the rational agents. How one models this wedge may thus matter for the robustness of the results. Chakraborty (2008), and Kobayashi and Inaba (2006) also point out that the BCA results for Japan are sensitive to how one models investment wedge. If it is modeled as a wedge on gross returns to capital instead, the results might change.

In the present paper, I choose to model labour market friction as an investment wedge instead of a capital wedge primarily because of two reasons. The first reason is methodological. The investment wedge directly identifies the matching probability \( q_t \) via the Tobin’s \( Q \) equation of the worker while the capital wedge does not.¹⁸ This approach to understand labour market friction using asset pricing principle is novel in the literature and demonstrates a convenient marriage between labour economics and finance. The second reason is that my formulation is data friendly. The matching probability can be directly estimated from the available macro data.

The issue still remains whether the key prediction of the model that labour market friction is pro-cyclical holds up with an alternative capital-wedge view of the friction. I explore this issue in the Appendix B by working out a perfect foresight version of the present model by replacing the investment wedge \( q_t \) by human capital wedge \( \tau_t^N \).

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¹⁸ To see this point clearly, note that the marginal return to human capital investment (recruitment) is \( \frac{\partial N_{t+1}}{\partial X_t} = \frac{q_t}{a} \) based on (2) and (4). The marginal return to physical capital investment \( \frac{\partial K_{t+1}}{\partial I_t} \) based on (3) is 1. The marginal rate of transformation between physical capital and human capital (which is the ratio of these two respective marginal returns) uniquely identifies the skill match probability \( q_t \) which is our key indicator of labour market friction. Such an identification of the skill matching probability may not be possible if the labour market friction is viewed as a human capital wedge.
The pro-cyclical behaviour of the friction with respect to the TFP is reasonably robust for plausible parametric restriction. 19

5. Conclusion
There is no consensus whether the labour market friction has increased or decreased in the US economy over the last few decades. The conventional Beveridge curve based explanation of labour market friction is problematic because of the remarkable volatility of the vacancy/unemployment ratio. In this paper, I take an asset pricing approach to understand the labour market friction. The labour market friction is viewed as an implicit tax on a firm’s investment in human capital. The model suggests that this wedge is inversely related to the firm’s recruitment productivity which in turn depends on the fundamental TFP shock driving the economy. In a booming economy, the recruitment productivity of the firm is lower due to tight labour market conditions. As a result, the incumbent workers enjoy a rent in terms of a higher Tobin’s Q. Viewed from this perspective, I conclude that the labour market friction gets aggravated during a period of TFP boom suggesting that the firm compromises the quality of the match in a tighter labour market condition. This pro-cyclical behaviour of the labour market friction is reasonably robust with respect to alternative formulation of labour market friction such as a human capital wedge. The model receives some empirical support from the labour market experiences of the US and Australia.

The public policy implication is that the government may need to invest more resources in job training program during a period of productivity boom when acute skill shortages could arise. A wide range of vocational courses can be set up at adult skill-centers to avoid these skill shortages. This would lower the Tobin’s Q of an employed worker and eliminate the rent that an incumbent worker receives during a productivity boom. A useful extension of this paper would be to model such an optimal education policy.

19 A comprehensive analysis of this robustness issue also involves an analysis with alternative stochastic specification of TFP as well as model specification, which is beyond the scope of this paper. The present model does not include a household sector. In a future paper, I plan to examine this issue using a dynamic stochastic general equilibrium model with various adjustment costs.
Appendix A

Derivation of Equation 19

Plug (17) into (15) to obtain

\[ aq_t^{-1} = A\epsilon_t^{1/(1-\alpha)} + .5\rho(1-\psi)E_t aq_{t+1}^{-1} \]  \hspace{1cm} (A.1)

where

\[ A = \left[ 5\rho(1-\alpha)\mu_1^{1/(1-\alpha)} \left\{ \frac{\alpha\rho}{1-\rho(1-\delta)} \right\}\right]^{\alpha/(1-\alpha)} \]

Conjecture a solution

\[ aq_t^{-1} = \Omega\epsilon_t^{1/(1-\alpha)} \]  \hspace{1cm} (A.2)

where \( \Omega \) is a coefficient to be determined by the method of undetermined coefficient.

Upon substitution in (A.1) and using the geometric lognormal random walk property of the TFP process \( \{\epsilon_t\} \) one obtains:

\[ \Omega\epsilon_t^{1/(1-\alpha)} = A\epsilon_t^{1/(1-\alpha)} + .5\rho(1-\psi)\mu_1^{1/(1-\alpha)}\Omega\epsilon_t^{1/(1-\alpha)} \]  \hspace{1cm} (A.3)

One can now uniquely solve \( \Omega \) which gives (20) and confirms that the conjecture (A.2) is right. This proves (19) and (20). //

Appendix B

Case when labour market friction is modeled as a human capital wedge

Let a positive fraction \( \tau^N_t \) represent the proportional wedge on the gross return on human capital. In a similar vein as in Kobayashi and Inaba (2006), I assume perfect foresight in the sense that the sequences of the human capital wedge, \( \{\tau^N_t\} \) and the TFP, \( \{\epsilon_t\} \) are known to the firm at date 0. The representative firm’s problem (P) now changes to:

\[ \text{Max } \sum_{t=0}^{\infty} \rho^t \left[ \epsilon_t K_t^\alpha N_t^{1-\alpha} - w_t N_t - \tau^N_t (MPN_t + 1 - \psi) N_t - X_t - I_t \right] \]

s.t. (2), (3) and (4).
Since the investment wedge is replaced by the capital wedge, set $q_t$ equal to unity. For simplicity, set the posting cost $a$ equal to unity. The TFP $\{\varepsilon_t\}$ is assumed to be exogenously given. The human capital wedge, $\tau_t^N$ is endogenous. The problem is to uncover the relationship between TFP and the human capital wedge sequence $\{\tau_t^N\}$. In other words, one is interested in signing the derivative $\frac{\partial \tau_t^N}{\partial \varepsilon_t}$.

The first order conditions are:

\[ I_t : \quad 1 = \rho(1 - (1 - \alpha)\tau_{t+1}^N)ak_{t+1}^{\alpha-1} + 1 - \delta \] 
\[ X_t : \quad 1 = \rho(1 - (1 - \alpha)\tau_{t+1}^N)(1 - \alpha)k_{t+1}^\alpha - w_{t+1} + (1 - \psi)(1 - \tau_{t+1}^N) \]

The wage bargaining equation (16) changes to:

\[ w_t = \text{arg max}[(1 - \tau_t^N)(MPN_t + (1 - \psi)) - w_t]^{\phi}(w_t - b_t)^{1-\phi} \]

which yields the following wage equation:

\[ w_t = (1 - \phi)(1 - \tau_t^N)(1 - \alpha)k_t^\alpha \varepsilon_t + 1 - \psi \]

which upon substitution in (B.2) gives:

\[ 1 = \rho[(\phi(1 - \tau_{t+1}^N) + \alpha \tau_{t+1}^N)(1 - \alpha)k_{t+1}^\alpha \varepsilon_{t+1} + \phi(1 - \psi)(1 - \tau_{t+1}^N)] \]

From (B.1) solve $k_{t+1}$ as:

\[ k_{t+1} = \left[ \frac{\alpha \rho(1 - (1 - \alpha)\tau_{t+1}^N)\varepsilon_{t+1}}{1 - \rho(1 - \delta)} \right]^{1-\alpha} \]
The model is thus summarized by two equations, (B.5) and (B.6) with two endogenous variables $k_{t+1}$ and $\tau_{t+1}^N$, and one exogenous variable $\varepsilon_{t+1}$. Based on (B.6), it follows that $\varepsilon_{t+1}$ impacts $k_{t+1}$ directly as well as indirectly via its effect on $\tau_{t+1}^N$. Define the right hand side of (B.6) as a function $h(\cdot)$. Thus (B.6) can be written in a compact form as follows:

$$k_{t+1} = h(\varepsilon_{t+1}, \tau_{t+1}^N(\varepsilon_{t+1})) \quad \text{(B.7)}$$

One can thus write the following derivative based on (B.7) using the implicit function theorem:

$$\frac{dk_{t+1}}{d\varepsilon_{t+1}} = \frac{\partial h}{\partial \varepsilon_{t+1}} + \frac{\partial h}{\partial \tau_{t+1}^N} \cdot \frac{\partial \tau_{t+1}^N}{\partial \varepsilon_{t+1}} \quad \text{(B.8)}$$

From (B.6) verify that the right hand side terms in (B.8), $\frac{\partial h}{\partial \varepsilon_{t+1}} > 0$ and $\frac{\partial h}{\partial \tau_{t+1}^N} < 0$.

To determine the sign of $\frac{\partial \tau_{t+1}^N}{\partial \varepsilon_{t+1}}$, differentiate (B.5) and employ (B.8) and obtain:

$$\frac{\partial \tau_{t+1}^N}{\partial \varepsilon_{t+1}} = \frac{\phi(1-\tau_{t+1}^N) + \alpha \tau_{t+1}^N k_{t+1}^\alpha}{\Delta} \quad \text{(B.9)}$$

where

$$\Delta = [(\phi - \alpha)(1-\alpha)\varepsilon_{t+1}k_{t+1}^\alpha + (1-\phi)(1-\psi)]$$

$$-\alpha(1-\alpha)\varepsilon_{t+1}k_{t+1}^{\alpha-1} \frac{\partial h}{\partial \tau_{t+1}^N} \cdot \{(1-\phi)(1-\tau_{t+1}^N) + \alpha \tau_{t+1}^N\} \quad \text{(B.10)}$$
Note that the numerator of (B.9) is positive. The second term of the denominator $\Delta$ is positive because $\frac{\partial h}{\partial \tau_{t+1}^N} < 0$ from (B.6). A sufficient condition for the first term of the denominator to be positive is that $\alpha < \phi$. Thus if $\alpha < \phi$, $\frac{\partial \tau_{t+1}^N}{\partial \varepsilon_{t+1}} > 0$.

A sufficient condition for the human capital wedge to be pro-cyclical with respect to the TFP is, therefore, $\alpha < \phi$. The intuition for this result is similar as in the case of an investment wedge. A higher prospective TFP translates into a higher capital:labour ratio which drives down the marginal product of tangible capital, $\text{MPK}$ and boosts the marginal product of intangible capital, $\text{MPN}$. If workers have weak bargaining power vis-à-vis firms in the sense that their share of the surplus $1 - \phi$ is less than their what they contribute to total output $(1 - \alpha)$, they pay a greater wedge. It is difficult to get a reliable estimate of the rent sharing parameter $\phi$. Mumford and Dowrick (1994) point out the methodological problem in estimating this parameter. Their estimate based on coal industry of New South Wales, Australia suggests that the estimate of worker’s rent sharing is about 10% which has to be interpreted with caution. Given this caveat and also given that the US capital share in GDP is about 0.36, the condition $\alpha < \phi$ appears like a plausible restriction.
Figure 1: Relative Price of Investment Goods with respect to Consumption Goods

Note: The relative price of investment goods with respect to consumer goods was constructed by computing the ratio of producer price indices of finished capital goods to the CPI. Both these monthly series came from US Department of Labour, BLS sources. Monthly numbers were converted to annual averages. Year 1992 is used as the base.
Figure 2: Relative Price of a Worker in terms of Capital Goods

Note: The relative price of worker with respect to capital is the ratio of the annual index of compensation per worker to the producer price index of finished capital goods over the period 1948-2001 taking 1992 as the base year. Data for compensation per worker came from Hall (2005) who compiled these data from Bureau of Labour Statistics (BLS). See note to Figure 1 for the source of the producer price index of finished capital goods.
Note: Actual Tobin’s Q of a worker is the trend adjusted relative price of worker. The TFP series is the Solow residual in Figure 4(a) and the Manufacturing multifactor productivity index. Model Tobin’s Q is based on equation (19).
Note: Same as in Figure 4. Matching Probability is q_t based on equation (19).
References


