

$$a) J_{(bound)} = \frac{\partial P}{\partial t} = \frac{\partial}{\partial t}(\epsilon_0 \chi E) = \epsilon_0 \chi j \times 1000\pi \times 3.5 e^{j\omega t}$$

(In this case  $\omega = 2\pi \times 500 \text{ s}^{-1}$ , 500 Hz oscillation.)

$$\epsilon^* = \epsilon' - j\epsilon'' = 1 + \chi, \chi = \epsilon^* - 1 = 6.94 - 2.51j$$

The amplitude of the current density is then:

$$|\epsilon_0 \times (6.94 - 2.51j) \times 1000\pi \times 3.5| = 8.85 \times 10^{-12} \times 7.38 \times 1000\pi \times 3.5 = \underline{\underline{7.18 \times 10^{-7} \text{ Am}^{-2}}} \text{ [2 marks]}$$

$$b) \text{ The current amplitude is } 7.18 \times 10^{-7} \times \text{area} = 7.18 \times 10^{-7} \times 25 \times 10^{-4} = \underline{\underline{1.80 \times 10^{-9} \text{ A}}} \text{ [2 marks]}$$

$$c) R = \frac{1}{\omega \epsilon'' C_0} = \frac{d}{\omega \epsilon'' \epsilon_0 A} = \frac{2 \times 10^{-3}}{1000\pi \times 2.51 \times 8.85 \times 10^{-12} \times 25 \times 10^{-4}} \\ = \underline{\underline{1.15 \times 10^7 \Omega}} \text{ [2 marks]}$$

$$d) \text{ By definition, the loss tangent is just } \tan \delta = \frac{\epsilon''}{\epsilon'} = \frac{2.51}{7.94} = \underline{\underline{0.316}} \text{ [2 marks]}$$

$$e) \text{ The power dissipation is given by either } W = \frac{1}{2} \text{Re}[I^* V] \text{ (longer approach), or,}$$

$$W = \frac{V_0^2}{2R}$$

where  $V_0$  is the amplitude of the voltage across the slab,  $V_0 = E_0 d$  where  $E_0 = 3.5 \text{ Vm}^{-1}$

$$\therefore V_0 = 3.5 \times 2 \times 10^{-3} \text{ V.}$$

$$\text{Thus } W = \frac{(3.5 \times 2 \times 10^{-3})^2}{2 \times 1.146 \times 10^7} = \underline{\underline{2.14 \times 10^{-12} \text{ W}}} \text{ [2 marks]}$$