Environmental Policy and Interjurisdictional Competition in a Second-Best World*

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Abstract: A common result in models of tax competition is that environmental quality is enhanced if jurisdictions cooperate over environmental policy. This paper challenges the standard view by showing that in a two-period model of tax competition, where saving behaviour is taken into account and distortionary taxes are present (second-best world), interjurisdictional competition may be desirable. Our analysis suggest that environmental policy may be stricter when jurisdictions are left free to compete over taxes on capital and pollution.

Keywords: Environmental policy, tax competition, distortionary taxation, savings.

JEL Classification: H2, H7

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1. Introduction

An ongoing debate in policy circles concerns the implications for environmental standards of enhanced economic integration and movements of factors of production across countries. During recent NAFTA negotiations, for example, the issue of a “race to the bottom” in environmental standards appeared to be of prominent importance. Indeed, a standard result in environmental economics is that jurisdictions should cooperate over environmental standards, if they wish to protect the environment. Cooperation will give rise to higher provision of public goods and higher environmental taxes or standards. If jurisdictions compete, a race to the bottom in environmental standards may occur in order to enhance international competitiveness and attract foreign capital. Cooperation will ensure that, for example, higher environmental taxes do result in higher environmental quality without the drawback of losing positions on the international arena. As Wilson (1997) points out, the fear of a race to the bottom has recently induced American politicians to shift the control of environmental policy from the States to the Federal government. List and Gerking (2000) however, do not find any evidence of a race to the bottom in environmental standards when US environmental policy was delegated to the States during President Regan era.

In fact, both the empirical evidence (see the survey of Levinson (1997) and the theoretical literature (see Wilson (1997) for a review) are inconclusive in whether a race to the bottom occurs.

An early paper on interjurisdictional competition and environmental policy which questions the occurrence of a race to the bottom is Oates and Schawb (1988). In this model, jurisdictions try to attract capital by competing over capital taxes and pollution standards. The optimal policy is to impose a capital tax equal to zero (since capital can escape abroad) and an environmental standard coinciding with the Pigovian level. The reason for the environmental standard result is that the government is using the environmental tax to correct for environmental externalities and not for other purposes such as revenue raising or public goods provision.

In an extension, they develop an endogenous policy model in which individuals are distinguished in wage and non-wage earners and the median voter takes decisions over a capital tax and a standard for local environmental policy. If the decisive individual is a wage earner, she will choose a negative capital tax and a higher environmental standard than the first-best optimal level. If the decisive individual is a non-wage earner, she will clearly prefer a positive tax (for redistributive reasons). However, in this case, whether the environmental standard is higher or lower than the first-best optimum is not clear cut.

The robustness of the Oates and Schawb’s results has been scrutinised by Wilson (1997). He reviews the role of capital taxes, unemployment and imperfect competition, among others, for the existence of a race to the bottom and draws the conclusion that the Oates and Schawb results are not conclusive for the race to the bottom debate.

Kim and Wilson (1997) present a second-best model of tax competition and the environment. In addition to the capital tax and the environmental standard, they include a distortionary labour tax and public goods provision. They find that in equilibrium, the capital tax is equal to zero, and the environmental standard is below the Pigovian level. Differently, from Oates and Schawb (1988), in this model there are two distortions: one caused by pollution externalities and the other by the labour tax. Revenue from the labour tax is therefore not enough for public goods provision. By relaxing the environmental standards, more capital can be attracted from abroad, which in turn increases the base of the capital tax and raises enough revenue for financing public goods. Kanbur, Keen, van Wijnbergen (1995) also find that countries will reduce their environmental standards below the Pigovian level in order to attract foreign investors.
It is important to notice that in the papers mentioned above, although a capital tax is introduced in the picture, how this tax influences saving behaviour is not considered (capital is held fixed in supply). The focus of these papers is essentially on the spatial dimension (that is tax competition across countries within a single time period) and not on the intertemporal dimension (which is tax competition when saving is endogenised in a dynamic economy framework).

In our review of pre-existing studies on tax competition and the environment we have abstracted from the extensive literature which focuses on plants location instead of capital movements (see in this respect, among others, Hoel (1997), Krumm and Wellisch (1995), Markusen et al. (1995), Motta and Thisse (1994), Rauscher (1995), Ulph (1994)), Venables (1999), Wellisch (1995)). The literature on plants location also abstracts from the issue of endogenous savings.

To our knowledge no model of environmental tax competition has considered savings behaviour.

Two related papers analyse savings behaviour under tax competition, Ha and Sibert (1997) and Klein, Rios-Rull, and Quadrini (2000), but they do not include environmental externalities or model government choice of public-goods provision. They generally find that tax competition may be desirable when governments cannot precommit to future taxes.

Our paper presents a two-period model of tax competition where saving behaviour is taken into account, and environmental policy considerations are present (in our paper, the government uses a pollution tax with the purpose of correcting for environmental externalities and a distortory capital tax to raise revenue for public goods provision). In doing so, it fills the gap between the static approach taken by the literature on tax competition and the environment and the dynamic framework adopted by the literature on interjurisdictional competition in capital taxation. In addition, we underline the relevance of dynamic effects and in particular saving behaviour for the race to the bottom debate in environmental policy.

The reason why savings behaviour is important is because one cannot consider the capital stock being invariant with respect to the fiscal regimes. If countries cooperate in setting capital taxes and environmental taxes, even if capital is fixed at the time when the governments choose the taxes, the individuals would have predicted (under perfect foresight) the taxes to be imposed. If two regimes give rise to different taxes, then these two regimes will induce different capital stocks. Comparing regimes and keeping the same capital stock under both may give misleading conclusions.

Uncoordinated policy making (tax competition) will induce a lower capital tax than coordinated policy making. Individuals will therefore save more (if savings respond positively to the after-tax return), and the uncoordinated regime will have larger capital stocks. If environment is a normal good, a larger capital stock makes a country choosing larger environmental consumption (i.e. a tougher standard). On the other hand, in the uncoordinated regime, countries do not internalize international spillovers. This implies that if the international spillovers are small the uncoordinated equilibrium gives less pollution, and if the spillovers are large, the opposite is true. Our model suggests that if spillovers are small uncoordinated policy making is desirable.

As in Ha and Sibert (1997) and Klein, Rios-Rull, and Quadrini (2000), the reason is that tax competition partially solves the time-inconsistency problem in capital taxation. The paper is organized as follows: section 2 presents the model; sections 3 and 4 describe the non cooperative and cooperative equilibrium respectively. Section 5 introduces the issue of endogenous savings and section 6 concludes.
2. The economy

2.1 Assumptions

Our economy consists of two countries (i and j). Consumers are homogeneous and derive utility from consumption in first and second period, denoted \( c_1 \) and \( c_2 \), respectively, and disutility from aggregate pollution \( \pi \), as well as utility from a public good, \( g \). All characteristics of country \( i \) are assumed to hold for country \( j \) as well (i.e. we only consider the symmetric case). We model the simplest possible case, abstracting from labour supply.

2.1.1 Preferences

To gain analytical tractability we assume the utility function to be

\[
U(c^1_i, c^2_i, g_i, \pi) = \left( \frac{c^1_i}{1 - \sigma} \right)^{1-\sigma} + \beta \left( \frac{c^2_i}{1 - \sigma} + \varepsilon \frac{g_i}{1 - \sigma} - D(\pi) \right)
\]

where \( \sigma \) is a utility parameter and \( \beta \) is the discount factor.

2.1.2 Budget constraints

In the beginning of the first period, individual \( i \) (the individual living in country \( i \)) receives lump-sum wealth \( w_0 \), which may be used for period-1 consumption, domestic investment, \( k^i_i \), and investment abroad \( k^j_i \) (superscript refers to residence, and subscript to the country). In the second period the individual may choose to relocate capital across borders, but at a cost (quadratic in the amount of capital relocated). Denote by \( \tau^i_i \) the capital tax imposed by the government in country \( i \) on residents’ domestic capital, and by \( \tau^j_i \) the capital tax imposed by the government in country \( j \) on non-residents’ capital in country \( j \). It is convenient to define the after-tax returns as

\[
\rho^i_i = (1 - \tau^i_i)r_i
\]

(2)

\[
\rho^j_i = (1 - \tau^j_i)r_j
\]

(3)

and similarly for an individual living in \( j \)

\[
\rho^i_j = (1 - \tau^i_j)r_i
\]

(4)

\[
\rho^j_j = (1 - \tau^j_j)r_j
\]

(5)

where \( r_i \) and \( r_j \) is the interest rate in country \( i \) and \( j \), respectively. The individual in \( i \) receives after-tax return on the final location of capital: \( k^i_i \), \( k^j_i \). Consequently the individual’s budget constraints are

\[
c^1_i = w_0 - k^i_i - k^j_i
\]

(6)

\[
c^2_i = \rho^i_i k^i_i + \rho^j_i k^j_i - \frac{\gamma}{2} \cdot k^i_i - \frac{\gamma}{2} \cdot k^j_i - \frac{\gamma^2}{2}
\]

(7)

where \( \gamma \) is a cost parameter of moving capital. We will later on analyse equilibria as \( \gamma \) goes to zero.
2.1.3 Production
Production takes place in both countries using a Cobb-Douglas CRS technology in physical capital and pollution. Firm i hires physical capital, $k_i^j + k_i^j$, at the rental rate $r_i$, and pays $\tau_i^j$ to the government per unit of pollution, $x_i$.

$$ f(k_i^j + k_i^j, x_i) = A(k_i^j + k_i^j)^a(x_i)^{1-a} $$

2.1.4 Pollution spillovers
Aggregate pollution is the sum of pollution generated in country $i$ and a fraction $\nu$ of pollution generated in country $j$

$$ \overline{x}_i = x_i + \nu x_j $$

2.1.5 Government’s budget
The government finances public consumption $g_i$ through taxes on domestically owned capital allocated at home, foreign capital located in country $i$, and domestic emissions:

$$ g_i = \tau_i^i r_i k_i^i + \tau_i^j r_i k_i^j + \tau_i^j x_i $$

2.1.6 Timing of the Policy Game

Non-cooperative equilibrium

1. The representative individual’s in country $i, j$, decide on savings and investment allocations (i.e. $\overline{k}_i, \overline{k}_i, \overline{k}_j, \overline{k}_j$), rationally predicting future policy.
2. In period 2 government $i$ chooses $\tau_i^i, \tau_i^j$, and $x_i$, (taking $\tau_j^i, \tau_j^j, and x_j$ as given) so as to maximise utility of individual $i$, government $j$ chooses $\tau_j^i, \tau_j^j$, and $x_j$ (taking $\tau_i^i, \tau_i^j, and x_i$ as given) so as to maximise utility of individual $j$.
3. Individuals observing government decision may choose to relocate capital across borders, and then production takes place (by profit maximising firms).
4. Taxation, public spending, and consumption takes place.

Cooperative equilibrium
In cooperative equilibrium, stage 2 is replaced by

2’. In period 2 government $i$ and $j$ choose $\tau_i^i, \tau_i^j, \tau_j^i, \tau_j^j, x_i, and x_j$ so as to maximise the sum of utilities of individuals $i$ and $j$.

This is equivalent to Nash bargaining between the two countries with equal weights. We solve the model backwards, beginning with stage 3, since we need to find the individuals’ reaction functions (i.e. capital allocation decisions as functions of policy). We call stage 3 the second period economic equilibrium.

2.2 Second Period Economic Equilibrium
In the second period (after the two governments have set their taxes) individual $i$ maximises consumption by moving capital across borders optimally (equivalent to maximizing second-period utility). This gives the individuals optimal allocation of capital ex post the
governments’ decisions

\[ k_i^i = \overline{k}_i^i + \frac{\rho_i^i - \rho_j^j}{2\gamma} \]  

(11)

\[ k_j^i = \overline{k}_j^i + \frac{\rho_j^j - \rho_i^i}{2\gamma} \]  

(12)

Those allocations are taken as constraints by the governments. Notice that if the after-tax returns are equal, the individual does not move capital across borders in the second period.

Define aggregate capital employed in production in each country as

\[ k_i = k_i^i + k_i^j \]  

(13)

\[ k_j = k_j^j + k_j^i \]  

(14)

Furthermore, profit maximization by firms gives

\[ r_i = \alpha(k_i)^{a-1}x_i^{1-a} \]  

(15)

\[ r_j = \alpha(k_j)^{a-1}x_j^{1-a} \]  

(16)

\[ \tau_i^i = (1 - \alpha)(k_i)^a x_i^{-a} \]  

(17)

\[ \tau_j^i = (1 - \alpha)(k_j)^a x_j^{-a} \]  

(18)

implying we can solve for \( k_i^i, k_i^j, k_j^i, k_j^j, r_i, r_j \) as functions of policy \( (\tau_i^i, \tau_j^i, \tau_j^j, x_i, x_j) \) only (given \( \overline{k}_i^i, \overline{k}_i^j, \overline{k}_j^i, \overline{k}_j^j \)).

Notice that, given constant returns-to-scale production technology, and profit maximisation by price taking firms, the government’s budget can be rewritten as total production minus the after-tax returns to factor owners (domestic and foreign):

\[ g_i = A(k_i)^a (x_i)^{1-a} - (1 - \tau_i^i)r_i k_i^i - (1 - \tau_j^i)r_j k_j^i \]  

(19)

where also (13) has been used.

Equivalently, by adding and subtracting \( \tau^i r_i k_i^i \) and using (2), equation (19) becomes

\[ g_i = A(k_i)^a (x_i)^{1-a} - \rho_i^i k_i + (\tau_i^j - \tau_i^i)r_j k_j^i \]  

(20)

We will now analyse stage 2 when governments compete over taxes.
3. Second period non cooperative equilibrium

In a non-cooperative equilibrium countries simultaneously set their policy instruments. Country \( i \) sets \( \tau_i^*, \tau_i^i, \) and \( x_i \) (taking \( \tau_j^*, \tau_j^i, \) and \( x_j \) as given); country \( j \) sets \( \tau_j^*, \tau_j^j, \) and \( x_j \) (taking \( \tau_i^*, \tau_i^j, \) and \( x_i \) as given). Both countries realize the effects of policy on \( k_i^i, k_i^j, k_j^i, k_j^j, r_i, r_j \). Each country maximises utility of the representative individual in their own country. We will only consider the symmetric case (when both countries are identical).

In stage 2 the individuals savings- and allocation decisions \( \overline{k}_i, \overline{k}_i, \overline{k}_j, \overline{k}_j \) are taken as given. However, since the individuals can relocate capital in stage 3, governments must realize that their policy choice affects the final capital allocation and the interest rates (i.e. \( k_i^i, k_i^j, k_j^i, k_j^j, r_i, r_j \)). We therefore need to find the derivatives of \( k_i^i, k_i^j, k_j^i, k_j^j, r_i, r_j \) with respect to the policy variables. In particular we need to know the derivative of the after-tax returns \( \rho_i^i \) and \( \rho_j^i \) (since the individual’s indirect utility depends on these), and of \( k_i \) and \( \rho_i^i \) (since they enter the government’s budget (20)). This is done in Appendix A.

We now state the government’s problem. We only consider the second-period utility (since the first period is in the past and given). Thus, government in \( i \) maximises the welfare function

\[
W^i = \frac{(c^i_2)^{1-\sigma}}{1-\sigma} + \varepsilon \frac{(g_i)^{1-\sigma}}{1-\sigma} - D(\overline{x}_i)
\]

subject to own budget constraint and the individuals’ response in capital relocation.

We solve for the optimal policy, under the assumption that adjustment costs are “small” (i.e. taking the limit as \( \gamma \) goes to zero). We have the following result:

**Proposition 1** As \( \gamma \to 0 \) the stage-2 symmetric non-cooperative equilibrium, taking \( \overline{k}_i, \overline{k}_i, \overline{k}_j, \overline{k}_j \) as given, is characterized by \( \tau_i^* = \tau_i^i = \tau_j^* = \tau_j^j = \tau \), and

\[
\tau^* = \frac{1}{\varepsilon} D(\overline{x}) g^\sigma
\]

\[
s = \frac{k_i^i}{k_j^j} = \frac{k_i^j}{k_j^i} = \frac{1}{1-\alpha} \frac{1}{1-\tau}
\]

\[
1 - s = \frac{1}{\varepsilon} \left[ \frac{1-\alpha}{\alpha} (1 + s) \right]^\sigma
\]

\[
\frac{g}{c_2} = (\varepsilon)^{1/\alpha} (1 - s)^{1/\alpha}
\]

**Proof:** See Appendix B.

**Remarks:**
(i) The Samuelson Rule for the economy above is \( \frac{g}{c_2} = (\varepsilon)^{1/\sigma} \)

(ii) If \( \varepsilon \) larger than/equal to/smaller than \( \left[ \frac{1 - \alpha}{\alpha} \right]^{\sigma} \) the capital-income tax is positive/zero/negative and public goods are under provided/Samuelson rule/under provided. When \( \varepsilon \) (the preference parameter over public consumption) is small, tax revenue from the environmental tax may be enough (or more than enough) to cover for the desired public consumption. For example if \( \varepsilon = \left[ \frac{1 - \alpha}{\alpha} \right]^{\sigma} \), the environmental tax revenue from a first-best Pigovian tax is exactly enough to give the first-best Samuelson rule for public consumption. This special case gives us thus the first-best. If \( \varepsilon < \left[ \frac{1 - \alpha}{\alpha} \right]^{\sigma} \) there is too much environmental tax revenue, and if a Pigovian tax was implemented public consumption would be overprovided in comparison to private. Thus the first best is not attained here. We find the situation when \( \varepsilon > \left[ \frac{1 - \alpha}{\alpha} \right]^{\sigma} \) more plausible. This is the situation when tax revenue from taxation of externalities are not enough to cover for public goods provision. Then the governments have to rely on distortionary taxes.

(iii) Our model also pins down the allocation of capital (the foreign/domestic capital ratio). This ratio is larger the larger the preference parameter for public expenditure is. The reason is that when the preference parameter is large, governments need to resort to capital taxation to a greater extent. In an equilibrium, competition among governments are exploited (to bring the taxes down) by the larger allocation of capital in the foreign country.

4. Second period cooperative equilibrium

Since we work with identical countries, there is no issue of redistribution between the countries. They therefore choose policies so as to maximize the sum of their welfare measures. Again we solve the second-period problem (taking savings as given).

The problem is to maximize

\[
W^{coop} = \frac{(c_i^2)^{1-\sigma}}{1-\sigma} + \varepsilon \frac{(g_i)^{1-\sigma}}{1-\sigma} - D(x_i) + \frac{(c_j^2)^{1-\sigma}}{1-\sigma} + \varepsilon \frac{(g_j)^{1-\sigma}}{1-\sigma} - D(x_j)
\]

Proposition 2 As \( \gamma \rightarrow 0 \) the stage-2 symmetric cooperative equilibrium, taking \( \bar{k}_i, \bar{k}_i, \bar{k}_j, \bar{k}_j \) as given, is characterized by \( \tau_i^* = \tau_j^* = \tau_i = \tau_j = \tau \), and

\[
\tau^* = \frac{1 + \gamma}{\varepsilon} D'(\bar{x}) g^\sigma
\]

\[
1 - \tau = \frac{1}{\alpha(1 + \varepsilon^{1/\sigma})}
\]

\[
\frac{g}{c_2} = (\varepsilon)^{1/\sigma}
\]
**Proof:** See Appendix B.

A comparison between the cooperative (coop) and the uncooperative (comp) equilibrium, given \( k_i^i, \bar{k}_i^j, k_j^l, \bar{k}_j^j \) gives the following:

\[
\begin{align*}
(30) & \quad [D'(\pi)\pi]_{\text{comp}} > [D'(\pi)\pi]_{\text{coop}} \\
(31) & \quad \left[ \frac{g}{f} \right]_{\text{comp}} < \left[ \frac{g}{f} \right]_{\text{coop}} \\
(32) & \quad \tau_{\text{comp}} < \tau_{\text{coop}}
\end{align*}
\]

This comparison holds when the capital stock is assumed to be the same under both regimes. The result confirms the findings of the static literature. That is, pollution in absolute value is larger under non-cooperation than under cooperation, and environmental strictness (in terms of pollution’s marginal product) is lower under non cooperation. At the same time the capital tax is lower under non-cooperation. However, if savings respond to the level of the capital tax the capital stocks will not coincide under both regimes. If individuals save less when the capital tax is higher, then there will be a larger capital stock under non-cooperation. We endogenise savings in the next section.

5. **Endogenous savings**

We now turn to the intertemporal problem. The equilibria in the two sections above are still valid (since savings are fixed in the second period), but now we endogenise the level of the capital stock. Given the regime the individuals can anticipate the capital tax, and the equilibrium interest rate in the second period. The individual in country i choose \( k_i^i + k_j^j \) so as to maximise the utility function above. The optimal savings decision is then

\[
(33) \quad \frac{\beta + \rho^{\frac{1-\sigma}{\sigma}}}{1 + \beta + \rho^{\frac{1-\sigma}{\sigma}}} w_0
\]

If \( \sigma \) is smaller than unity savings respond positively with an increase in the after tax return \( \rho \). Then there is the possibility that, even though the absolute value of emissions is larger in the non-cooperative regime, the capital/emissions ratio may be larger. Since there is a one-to-one relation between the capital/emissions ratio and emissions marginal product, we may have the marginal product of emissions greater in the non-cooperative regime, i.e. environmental policy may be stricter under non cooperation.

Finally, if pollution travels across borders, cooperation is more likely to be desirable. One need to evaluate the effect of internalising the externality across borders against the efficiency gain in capital taxation.

**Proposition 3** Let \( \gamma \rightarrow 0 \), assume \( \sigma < 1 \), and \( D(\pi) = \eta \pi \), and let \( s \) be the solution to (24). Then, if (and only if):

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there exists a parameter configuration for which the environmental tax is higher in non-cooperative equilibrium than in the cooperative equilibrium: $\tau^{\text{comp}} > \tau^{\text{coop}}$.

**Proof:** See Appendix C.

We see from (34) that it is necessary that $v$ is small enough relative to $\varepsilon$ (otherwise the right-hand side is negative) for having $\tau^{\text{comp}} > \tau^{\text{coop}}$. The reason is that if spillovers are large the environmental tax is larger (everything else equal) under cooperation, since the spillover is not taken into account by countries under competition. If $\varepsilon$ is large, the second-best constraint is tighter (the government taxes more to provide the public good). Tightening the second-best constraint makes the time-inconsistency problem more severe, and there is more gain from tax competition.

If savings is responsive enough to the capital tax, and international pollution spillovers are small, the lower capital tax induces more savings and increases the affordability of environmental policy. This suggests that welfare may as well be greater under tax competition than under cooperation. This is because competition acts as a partial substitute for commitment.

We will now provide numerical examples. We choose parameter values that give a bench-mark equilibrium reasonably consistent with actual economies in Europe. The purpose is not to match actual economies (we would need a richer model for that) but to illustrate which of the regimes is more likely to give higher pollution taxes.

Since European countries do not cooperate in taxation (though there are proposals to do so in the future), the bench-mark case is non-cooperation. We choose $\tau^{\text{comp}} = 0.15$, as a cross-country average effective capital tax rate (according to King and Fullerton the range is between 0.09 and 0.19).

We think of $x$ as energy causing pollution. For the bench-mark we take the petrol tax in the UK, see IEA (2001), giving $\tau^x = 0.75$. We also set the share of $x$ in production to 0.2, implying we choose $\alpha = 0.8$. The discount factor is computed as follows. Since we have two-period model, we think of each period as 20 years (i.e. 40 years of active economic age). We take the yearly discount rate to be 2.7% (see Krueger and Kubler, 2003). We set $\delta = 0.027^{20} = 0.59$ (this is lower than would have been used in a dynastic economy). Next we choose values for $\sigma$. There is a range of savings elasticities reported in the empirical literature, between 0 and 0.4. Gorter and de Mooij (2001) report that the compensated savings elasticity range from 0.2-0.4. We compute the corresponding values of $\sigma$, allowing for period 2 consumption being 40% greater than period one (to allow for a yearly consumption growth of 1.7%, see Krueger and Kubler, 2003), see Appendix D for details. $\sigma = 0.42$ corresponds to a compensated savings elasticity of 0.2 and $\sigma = 0.49$ corresponds to an elasticity of 0.4. In our computations the results vary negligibly when we vary $\sigma$ between 0.42 and 0.49. We therefore only report our results for $\sigma = 0.45$, corresponding to a compensated savings elasticity of 0.3. We choose initial wealth, $w_0$, so that period-2 consumption is unity, giving us $w_0 = 1.22$. We set $A$ consistent with the assumed pollution tax, implying $A = 3.05$. We set the preference parameter for public goods consistent with (25), giving $\varepsilon = 6.06$. Finally we set the preference parameter for pollution so that (22) holds, implying $\eta = 6.375$.

The bench-mark gives us the following non cooperative equilibrium.
Table 1. Non cooperative equilibrium (benchmark)

\[
\begin{align*}
  r &= 2.32 \text{ (corresponding to a yearly interest rate of 4.3\%)} \\
  k &= 0.507 \\
  s &= 0.88 \\
  g &= 0.47 \\
  y &= 1.47 \\
  x &= 0.66
\end{align*}
\]

This gives public spending as a fraction of GDP, \( \frac{g}{y} = 0.32 \).

We next compute the cooperative equilibrium, applying the same parameter values as above. This tells us, hypothetically, what would happen if the two countries were to cooperate. The equilibrium will depend on the pollution spillover parameter \( \nu \). EMEP (2001) reports percentages of own pollution and of foreign pollution for various countries. For example, Britain receives 20\% of oxidized sulphur from abroad while 80\% is self generated. This would imply a spillover parameter of \( \nu = 0.25 \). Norway, on the other extreme, receives 92\% from abroad, implying \( \nu = 12 \). We try values of \( \nu \) between 0 and 13.

First, the tax rate is independent of pollution spillovers, and is \( \tau^{\text{coop}} = 0.978 \) which is close to confiscatory. The individuals foresee this and save virtually nothing, implying close to zero consumption in period 2. Public goods consumption is also close to zero, even though the capital tax is very high. This is because the individuals rationally save very little. This indicates the severity of the time-inconsistency problem, and shows that tax competition (our bench mark \( \tau^{\text{comp}} = 0.15 \)) acts as substitute for commitment.

The pollution tax rate is lower for the entire range of the spillover parameter. We report some of the values below.

Table 2. Pollution tax under cooperation

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( \tau^{\nu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.058</td>
</tr>
<tr>
<td>0.2</td>
<td>0.062</td>
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</tbody>
</table>

Still, with very high spillovers, the pollution tax is much smaller than under tax competition. This implies that cooperation in fiscal policy does not, in our bench-mark model, increase the environmental tax. On the contrary it is quantitatively much lower.

To conclude, tax coordination is a very costly institutional arrangement, both in terms of private and public consumption.

6. Conclusions
We have addressed the issue of the effect of cooperation versus non-cooperation among two countries in second-best policy making. We found that when one takes the capital stock to be exogenous (and invariant across fiscal regimes), cooperation always produces tougher environmental protection. However when capital is endogenised (modelled as foregone consumption) the result may be reversed. The reason is that tax competition acts as a partial precommitment mechanism, and lowers the equilibrium capital tax. Individuals will save more in such a regime, and consequently the capital stock will be larger. The wealth effect from having a larger capital stock implies the economy optimally chooses to protect the environment more (measured in terms of a pollution tax). If pollution is transboundary, there may be gains from cooperation, since one can internalise the international externality. However, one has to compare this gain again the loss of efficiency in capital taxation, due to cooperation. We derived analytically a condition showing that when the spillovers are large, cooperation leads to a larger environmental tax.

To get an idea of which regime is likely to dominate when considering actual economies, we calibrated the model so that the non-cooperative equilibrium is quantitatively close to actual economies. We then computed the cooperative equilibrium and found that for the entire empirically relevant range of the spillover parameter, the environmental tax under competition is much larger (quantitatively) than under cooperation. The reason is that the time-inconsistency problem is quantitatively severe (taxes under cooperation are very large) and competition serves as a substitute for commitment. The welfare consequences of cooperation are also severe. Both private and public consumption are at very low levels under cooperation.

Our results suggest that there may be gains from cooperating in one dimension only (environmental policy) and maintaining competition in capital taxation. This however has to be explored in a formalized model, because even though countries compete in capital taxes, the cooperation in environmental policy can act as an implicit capital tax manipulating capital’s marginal product. We leave this for further work.

References


Press.


APPENDIX A Proof of Proposition 1

We need to find the derivatives of $k_i^j, k_i^i, k_j^i, r_i, r_j$ with respect to the policy variables. In particular we need to know the derivative of the after-tax returns $\rho_i^1$ and $\rho_i^1$ (since the individual’s indirect utility depends on these), and of $k_i$ and $\rho_i^1$ (since they enter the government’s budget (20)). This is done in a series of steps below.

The specification of the production technology gives Lemma 1 below:.

**Lemma 1** From the viewpoint of government $i$ the following holds

\[(A.1) \quad \frac{dr_i}{\tau_i} = -(1 - \alpha) \frac{dk_i}{k_i} + (1 - \alpha) \frac{dx_i}{x_i}\]

\[(A.2) \quad \frac{dr_j}{\tau_j} = -(1 - \alpha) \frac{dk_j}{k_j}\]

*Proof:* Follows by taking the differential of (15) and (16), respectively. Notice that $x_j$ is taken as given by government $i$.

**Lemma 2** From the viewpoint of government $i$ the following holds

\[(A.3) \quad \frac{d\rho_i^j}{\rho_i^j} = \frac{dr_i}{\tau_i} - \frac{d\tau_i^j}{1 - \tau_i^j}\]

\[(A.4) \quad \frac{d\rho_j^i}{\rho_j^i} = \frac{d\rho_i^j}{\rho_i^j} = \frac{dr_j}{\tau_j}\]

\[(A.5) \quad \frac{d\rho_i^j}{\rho_i^j} = \frac{dr_i}{\tau_i} - \frac{d\tau_i^j}{1 - \tau_i^j}\]

*Proof:* Follows by taking the differential of (2)-(5).

We should notice that the derivatives need to be evaluated at the equilibrium. Since we model identical countries we look at the symmetric equilibrium, where the after-tax returns as well as the capital stocks and pollution are equalized across countries. We also take the limit of the derivatives as $\gamma \to 0$.

**Lemma 3** From the viewpoint of government $i$, at a symmetric equilibrium, the following holds

\[(A.6) \quad \frac{dk_i}{k} \bigg|_{\gamma=0} = \frac{1}{2} \frac{dx_i}{x} - \frac{1}{4} \frac{d\tau_i^1 + d\tau_i^1}{(1 - \alpha)(1 - \tau)}\]
\[ \frac{dr_i}{r_i} \big|_{\gamma=0} = \frac{1}{4} \frac{d\tau_i^j + d\tau_j^i}{1 - \tau} + \frac{1 - a}{2} \frac{dx_i}{x} \]

\[ \frac{dr_j}{r_j} \big|_{\gamma=0} = -\frac{1}{4} \frac{d\tau_i^j + d\tau_j^i}{1 - \tau} + \frac{1 - a}{2} \frac{dx_i}{x} \]

**Proof:** Evaluating (12) for the foreign investor and adding to (12) gives the total capital in \( i \) as a function of the after-tax returns:

\[ k_i = \overline{k}_i^i + \overline{k}_i^j + \frac{\rho_j^i + \rho_j^i - \rho_i^j - \rho_i^j}{2\gamma} \]

The differential is

\[ 2\gamma dk_i = dp_i^j + dp_i^i - dp_j^i - dp_j^i = \left( \rho_i^j + \rho_i^j \right) \frac{dr_i}{r_i} - \left( \rho_j^i + \rho_j^i \right) \frac{dr_j}{r_j} - \rho_i^j \frac{dx_i}{x_i} - \rho_i^j \frac{dx_i}{x_i} \]

where the last equality follows from Lemma 2. Evaluating (A.10) at the symmetric equilibrium we have

\[ \frac{\gamma}{\rho} dk_i = \frac{dr_i}{r_i} - \frac{dr_j}{r_j} - \frac{1}{2} \frac{d\tau_i^i + d\tau_j^i}{1 - \tau} \]

Using Lemma 1 we obtain

\[ \frac{\gamma}{\rho} dk_i = -(1 - a) \frac{dk_i}{k_i} + (1 - a) \frac{dx_i}{x_i} + (1 - a) \frac{dk_j}{k_j} - \frac{1}{2} \frac{d\tau_i^i + d\tau_j^i}{1 - \tau} \]

Since \( k_j = \overline{k}_i^j + \overline{k}_j^j + \overline{k}_j^i - k_i \), we have \( dk_j = -dk_i \), then we obtain

\[ \left[ \frac{\gamma}{\rho} + \frac{1 - a}{k_i} + \frac{1 - a}{k_j} \right] dk_i = (1 - a) \frac{dx_i}{x_i} - \frac{1}{2} \frac{d\tau_i^i + d\tau_j^i}{1 - \tau} \]

Taking limit \( \gamma \to 0 \) (N.B.\( k_i = k_j = k \) in the symmetric equilibrium) gives (A.6). Inserting (A.6) into (A.1) gives (A.7). Using \( dk_j = -dk_i \) and inserting (A.6) into (A.2) gives (A.8).

QED

**Lemma 4** From the viewpoint of government \( i \), at a symmetric equilibrium, the following holds

\[ \frac{dr_i}{r_i} \big|_{\gamma=0} = -\frac{3}{4} r d\tau_i^j + \frac{r}{4} d\tau_j^i + \frac{1 - a}{2} \rho_i \frac{dx_i}{x_i} \]
Proof: Follows by substituting (A.7) and (A.8) from Lemma 3 into Lemma 2.

**Lemma 5** From the viewpoint of government $i$, at a symmetric equilibrium, the following holds

\[(A.17) \quad \frac{dg_i}{rk} \mid_{\gamma=0} = \frac{2-a}{2a} (1-a+at) \frac{dx_i}{x} + \left[ \frac{3}{4} - \frac{a}{4(1-a)(1-t)} - \frac{k_i}{k} \right] d\tau_i^j + \left[ \frac{k_i}{k} - \frac{a}{4(1-a)(1-t)} - \frac{1}{4} \right] d\tau_i^j \]

**Proof:** The differential of (20) is

\[(A.18) \quad dg_i = [aA\left(\frac{k_i}{x_i}\right)^{a-1} - \rho_j]dk_i + (1-a)A\left(\frac{k_i}{x_i}\right)^a dx_i - k_i d\rho_j + d[(\tau_i^j - \tau_i^j)r, k_i^j] \]

or by using the properties of the production function

\[(A.19) \quad dg_i = (r_i - \rho_i')k_i \frac{dk_i}{k_i} + \frac{1-a}{a} r_i k_i \frac{dx_i}{x_i} - k_i d\rho_j + d[(\tau_i^j - \tau_i^j)r, k_i^j] \]

Notice that $(\tau_i^j - \tau_i^j)d(r, k_i^j) = 0$ because $\tau_i^j = \tau_i^j$ in the symmetric equilibrium when $\gamma = 0$. Next, using (A.6) from Lemma 3 and (A.14) from Lemma 4, and rearranging, gives (A.17).

QED

We are now in a position to prove Proposition 1. For convenience let $z = \langle \tau_i^j, \tau_i^j, x_i \rangle$.

Next, notice that $\frac{\partial c_i^j}{\partial \rho_i^j} = k_i$, $\frac{\partial c_i^j}{\partial \rho_j^i} = k_j$ due to the envelop condition. The FOCs to government $i$’s problem are

\[(A.20) \quad \frac{\partial W_i}{\partial z} = (c_i^j)^{-\sigma} \left( k_i^j \frac{\partial \rho_i^j}{\partial z} + k_j^i \frac{\partial \rho_j^i}{\partial z} \right) + \varepsilon(g_i)^{-\sigma} \frac{\partial g_i}{\partial z} - D'(x_i) \frac{\partial x_i}{\partial z} = 0 \]

First, notice that in a symmetric equilibrium $c_i^j = (\rho_i^j k_i^j + \rho_j^i k_j^i) = \rho(k_i^j + k_j^i) = \rho k_i = \rho k$.

Next, using (A.6), (A.14), (A.15), and (A.19) the first-order conditions for the government become
(A.21) \( \frac{\partial W_i}{\partial r_i} \mid_{\gamma=0} = (pk)^{-\sigma} \left( -\frac{3}{4} r k_i - \frac{r}{4} k'_i \right) + \varepsilon(g)^{-\sigma} r k \left[ \frac{3}{4} - \frac{\tau}{4(1-\alpha)(1-\tau)} - \frac{k'_i}{k} \right] = 0 \)

(A.22) \( \frac{\partial W_i}{\partial r_i} \mid_{\gamma=0} = (pk)^{-\sigma} \left( \frac{r}{4} k'_i - \frac{r}{4} k_i \right) + \varepsilon(g)^{-\sigma} r k \left[ \frac{k'_i}{k} - \frac{\tau}{4(1-\alpha)(1-\tau)} - \frac{1}{4} \right] = 0 \)

(A.23) \( \frac{\partial W_i}{\partial x_i} \mid_{\gamma=0} = (pk)^{-\sigma} \left( \frac{1-\alpha}{2} \frac{\rho}{x} k_i + \frac{1-\alpha}{2} \frac{\rho}{x} k'_i \right) + \varepsilon(g)^{-\sigma} \frac{2 - \alpha}{2a} (1 - \alpha a \tau) \frac{r k}{x} - D'(\tau) = 0 \)

Deduct (A.22) from (A.21), and divide by \( r \) then

(A.24) \( 0 = -(pk)^{-\sigma} k'_i + \varepsilon(g)^{-\sigma} \left[ k - 2k'_i \right] = -(pk)^{-\sigma} k'_i + \varepsilon(g)^{-\sigma} \left[ k'_i - k_i \right] \)

which gives

(A.25) \( (pk)^{-\sigma} = \varepsilon(g)^{-\sigma} \left[ 1 - k_i/k'_i \right] \)

Inserting (A.25) into (A.22) and multiply by \( 4/r \) gives

(A.26) \( 0 = \left[ 1 - k_i/k'_i \right] (k'_i - k_i) + \left[ 4k'_i - \frac{\tau k_i}{(1-\alpha)(1-\tau)} - k \right] \)

or

(A.27) \( 0 = \left[ 1 - k_i/k'_i \right] (k'_i - k_i) + \left[ 3k'_i - k_i - \frac{\tau(k'_i + k_i)}{(1-\alpha)(1-\tau)} \right] \)

Defining \( s = \frac{1}{1-\alpha} \frac{\tau}{1-\tau} \) and inserting into (A.27) gives

(A.28) \( 0 = \left[ 1 - k_i/k'_i \right] (k'_i - k_i) + \left[ 3k'_i - k_i - s(k'_i + k_i) \right] \)

which in turn gives \( k'_i = sk_i \). This proves (23) Next, using (23) in (A.25) gives (25).

To derive (22) we insert (A.25) into (A.23) to obtain

(A.29) \( \left[ 1 - k_i/k'_i \right] \cdot \frac{1-\alpha}{2} \frac{\rho}{x} k'_i + \frac{1-\alpha}{2} \frac{\rho}{x} k_i \cdot + \frac{2 - \alpha}{2a} (1 - \alpha a \tau) \frac{r k}{x} - \frac{1}{\varepsilon \sigma} g D'(\tau) = 0 \)

Then, using \( k_i = sk'_i \) we have

(A.30) \( \left[ 1 - s \right] \frac{(1-\alpha)(1-\tau)}{2} + \frac{2 - \alpha}{2a} (1 - \alpha a \tau) \frac{r k}{x} - \frac{1}{\varepsilon \sigma} g D'(\tau) = 0 \)

which further rearranged gives

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By the definition of \( s \) (A.31) becomes

\[
(A.32) \quad \frac{1 - a}{a} \frac{r k}{x} - \frac{1}{c} g^a D'(\pi) = 0
\]

which gives (22), since \( r k/x = \tau^*(1 - a)/a \).

Finally, evaluating (20) in equilibrium gives

\[
g = A(k)^a(x)^{1-a} - \rho k = r k / \alpha - \rho k = \rho k \left[ \frac{1}{\alpha (1 - \tau)} - 1 \right].
\]

Using \( s = \frac{1}{1 - a} - \frac{\tau}{1 - \tau} \) gives

\[
(A.33) \quad g = \rho k \frac{1 - a}{a} (1 + s)
\]

Inserting into (25), noticing that \( c_2 = \rho k \), gives (24). QED

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APPENDIX B Proof of Proposition 2

The global government controls all after tax prices. We need to optimize with respect to \( \rho_i, \rho_j, \rho_i, \rho_j, x_i, \text{ and } x_j \).

First we take some preliminary steps. Take the differential of (19) to obtain

\[
(B.1) \quad dg_i = r_i dk_i + \tau_i dx_i - k_i dp_i - k_i dp_i - \rho_i dk_i - \rho_i dk_i
\]

Take the differentials of (A.9), (11), and equation (12) corresponding to individual \( j \) and substitute into (B.1) to obtain

\[
(B.2) \quad dg_i = \frac{r_i - \rho_i}{2 \gamma} (dp_i - dp_i) + \frac{r_i - \rho_i}{2 \gamma} (dp_i - dp_i) + \tau_i dx_i - k_i dp_i - k_i dp_i
\]

This equation for country \( j \) is

\[
(B.3) \quad dg_j = \frac{r_j - \rho_j}{2 \gamma} (dp_j - dp_j) + \frac{r_j - \rho_j}{2 \gamma} (dp_j - dp_j) + \tau_j dx_j - k_j dp_j - k_j dp_j
\]

For convenience let \( z = \{ \rho_i, \rho_j, \rho_i, \rho_j, x_i, x_j \} \). As in the proof of Proposition 1,

\[
\frac{\partial c^2_x}{\partial \rho_i} = k_i, \quad \frac{\partial c^2_x}{\partial \rho_j} = k_j
\]

due to the envelop condition. The FOCs to the cooperative problem are

\[
\frac{\partial W_{coop}}{\partial z} = (c^2_z)^{-\sigma} k_i \frac{\partial \rho_i}{\partial z} + k_j \frac{\partial \rho_j}{\partial z} + (c^2_z)^{-\sigma} k_j \frac{\partial \rho_j}{\partial z} + k_j \frac{\partial \rho_j}{\partial z}
\]
\[ + \varepsilon(g_i)^{-\sigma} \frac{\partial g_i}{\partial z} + \varepsilon(g_j)^{-\sigma} \frac{\partial g_j}{\partial z} - D'(\bar{x}_i) \frac{\partial \bar{x}_i}{\partial z} - D'(\bar{x}_j) \frac{\partial \bar{x}_j}{\partial z} = 0 \]

In particular, for \( \rho_i \) we have, by using (B.2) and (B.3)

\[ \frac{\partial W_{coop}}{\partial \rho_i} = (c_i^\prime)^{-\sigma} \rho_i^\prime + \varepsilon(g_i)^{-\sigma} \left[ \frac{r_i - \rho_i^\prime - k_i^\prime}{2\gamma} \right] - \varepsilon(g_j)^{-\sigma} \frac{r_j - \rho_j^\prime}{2\gamma} = 0 \]

which evaluated at the symmetric equilibrium \( (g_i = g_j = g, r_i = r_j = r, \rho_i = \rho_j = \rho) \) gives (29).

The other first-order conditions, with respect to \( \rho_i, \rho_j, \) and \( \rho_j \), give the same result as above.

The first-order condition with respect to \( x_i \), by using (B.2) and (B.3), is

\[ \frac{\partial W_{coop}}{\partial x_i} = \varepsilon(g_i)^{-\sigma} \tau_i^\prime - D'(\bar{x}_i) - D'(\bar{x}_j) \nu = 0 \]

Evaluating (B.6) at the symmetric equilibrium \( (g_i = g_j = g, \bar{x}_i = \bar{x}_j = \bar{x}, \tau_i^\prime = \tau_j^\prime = \tau^\prime) \) gives (27).

The first-order condition with respect to \( x_j \) gives the same result.

Finally, evaluating (20) in equilibrium gives

\[ g = A(k)^{\alpha} (x)^{1-\alpha} - \rho k = r k^2 - \rho k = \rho k \left[ \frac{1}{a(1 - \tau)} - 1 \right] \]

Since \( c_i^\prime = \rho k \), we obtain (28). QED

APPENDIX C Proof of Proposition 3

We will prove the possibility for a linear damage function, so that \( D'(x) \) is a constant, say unity.

Denote the non cooperative equilibrium by \( * \). Substitute (25) into (22), and (29) into (27) respectively (and notice that \( c_i^\prime = \rho k \) in both regimes). Then we have

\[ \tau^{**} = (\rho^* k^*)^{\alpha} (1 - s) \]

\[ \tau^* = (\rho k)^{\alpha} (1 + \upsilon) \]

Take logs of both sides and take the differential, using (33),

\[ \frac{d\tau^*}{\tau^*} = \frac{d\rho}{\rho} \left[ \frac{1 + \sigma \beta \rho^{\frac{1}{\alpha}}}{1 + \beta \rho^{\frac{1}{\alpha}}} \right] + \frac{d\beta}{\beta} \left[ \frac{1 + \sigma \rho^{\frac{1}{\alpha}}}{1 + \beta \rho^{\frac{1}{\alpha}}} \right] \]
Since the capital tax is independent of $\beta$ we have $\frac{d\rho}{\beta} = \frac{dr}{r}$. Furthermore, by the production technology we have

\begin{equation}
\frac{dr}{r} = \frac{\alpha - 1}{\alpha} \frac{d\tau^x}{\tau^x}
\end{equation}

Then, after rearrangement

\begin{equation}
\frac{d\tau^x}{d\beta} = \frac{x^x}{\beta} \frac{1 + \sigma \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}}{1 + \left(\frac{1-\alpha}{\alpha}\sigma\right) \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}} > 0
\end{equation}

Differentiating the log of the right hand side with respect to $\rho$ gives

\begin{equation}
\frac{d}{d\rho} \ln \left( \frac{1 + \sigma \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}}{1 + \left(\frac{1-\alpha}{\alpha}\sigma\right) \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}} \right) = \frac{-\left(1 - \sigma\right)^2 \beta^\frac{1}{\alpha} \rho^\frac{1}{\alpha}}{\left(1 + \sigma \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}\right) \left[\frac{1}{\alpha} + \left(\frac{1-\alpha}{\alpha}\sigma\right) \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}\right]}
\end{equation}

Since there is a negative relationship between $\rho$ and the environmental tax (follows by (C.4) since $\tau$ is held fixed) the total of the right-hand-side is larger for larger environmental tax. Thus, if we find a $\beta_0$ for which the environmental taxes coincide across regimes, then by picking a $\beta$ lower than $\beta_0$ implies that the environmental tax is greater in the non-cooperative equilibrium.

We will now show that such a $\beta_0$ exists for certain parameter values. Then there is a range of values for $\beta : \beta < \beta_0$ and consequently a parameter range for which the non-cooperative equilibrium gives rise to a higher environmental tax. Divide (C.2) by (C.1)

\begin{equation}
\frac{\tau^x}{\tau^{x*}} = \left(\frac{\rho k^*}{\rho^* k^*}\right)^\sigma \frac{1 + \nu}{1 - s}
\end{equation}

Whenever $\beta_0$ exists both left-hand and right-hand sides equal unity for $\beta_0$. When it doesn’t exist and RHS is greater/smaller than unity the cooperative regime delivers higher/lower environmental tax than the non-cooperative.

Assuming such a $\beta_0$ exists, (C.7) may, by using (33), be written as

\begin{equation}
1 = \frac{\rho}{\rho^*} \left(\frac{1 + \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}}{1 + \beta^\frac{1}{\alpha} \rho^\frac{1+\sigma}{\alpha}} \right)^\sigma \frac{1 + \nu}{1 - s}
\end{equation}

Since $\tau^{x*} = \tau^x$, then $r^* = r$. Then we have
\[(C.9) \quad 1 = \frac{1 - \tau}{1 - \tau^*} \left( \frac{1 + m(1 - \tau^*)^{\frac{1}{\alpha}}}{1 + m(1 - \tau)^{\frac{1}{\alpha}}} \right)^{\frac{\sigma}{1 - s}} \]

where \( m = \beta^\frac{1}{\alpha} \frac{1}{\delta}. \)

Since \( A \) can be chosen freely in the production technology, choosing \( \beta \) is equivalent to choosing \( m \). We need to show that there is an \( m \) (between zero and infinite) satisfying (C.9). The right-hand side of (C.9) is increasing in \( m \) since \( \sigma < 1 \) and \( \tau^* < \tau \) (can be shown by differentiating (C.9)), and consequently reaches minimum at \( m = 0 \) (we cannot allow \( m = 0 \) in the model, but we now just examine the borders of the parameter values) and a maximum at \( m \) going to infinity. Therefore, the maximum must exceed unity and the minimum fall below unity, i.e.

\[(C.10) \quad \frac{1 - \tau}{1 - \tau^*} \frac{1 + \nu}{1 - s} < 1 < \left( \frac{1 - \tau}{1 - \tau^*} \right)^{\frac{\sigma}{1 - s}} \frac{1 + \nu}{1 - s} \]

However, if the maximum level is smaller than unity then there exists no \( \beta_0 \) so for whatever \( \beta \) being assumed in the model the right-hand side falls below unity and the environmental tax is greater under non-cooperation. Similarly, if the minimum level is greater than unity there is no \( \beta_0 \) so for any \( \beta \) being assumed in the model, the right-hand side exceeds unity and the environmental tax is greater under cooperation.

Whenever

\[(C.11) \quad \frac{1 - \tau}{1 - \tau^*} \frac{1 + \nu}{1 - s} < 1 \]

holds, there is always a \( \beta \) in an interval (smaller than \( \beta_0 \), if it exists) that gives a higher environmental tax under non-cooperation. Now, using (23) to substitute for \( \tau^* \) and (28) to substitute for \( \tau \) in (C.11) gives

\[(C.12) \quad \frac{1 + (1 - \alpha)s}{\alpha(1 + e^{\frac{1}{\alpha}})} \frac{1 + \nu}{1 - s} < 1 \]

rearranging gives (34). One can easily find parameters such that (34) holds. For example, let \( \sigma = 1/2, \alpha = 6/11, \epsilon = 5/4 \), then \( s = 1/5 \) (by solving (24)), and (34) holds if \( \nu < 1/40 \). QED

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**APPENDIX D Compensated savings elasticity**

Differentiating (33) with respect to \( \rho \) and allowing \( w_0 \) to change so as to keep utility constant gives the compensated elasticity
Denote the consumption growth rate as \( h \), then from the individual’s budget
\[
(D.2) \quad \rho k = h(w_0 - k)
\]
Combining with (33) and rearranging gives
\[
(D.3) \quad \beta \frac{1}{h} \rho = h
\]
Inserting (D.3) into (D.1) gives
\[
(D.4) \quad \frac{\rho}{k} \frac{dk}{d\rho} = \frac{1}{\sigma(1 + \beta h^{(1-\sigma)})} - 1
\]
For given \( \beta \) and \( h \) this gives \( \sigma \) as a function of the compensated elasticity.

REFERENCE’S APPENDIX
Deriving (A.17) from (A.19)

Equation (A.19) may be written as
\[
(R.1) \quad \frac{d\xi}{rk} \bigg|_{\gamma=0} = \frac{r-\rho}{r} \left[ \frac{1}{2} \frac{dx_i}{x} - \frac{1}{4} \frac{d\tau^i}{x} + \frac{1}{4} \frac{d\tau^i}{x} + \frac{1}{4} \frac{d\tau^i}{x} \right] + \left[ \frac{3}{4} - \frac{r}{4r(1-\alpha)(1-\tau)} \right] d\tau^i
\]
Collecting terms gives
\[
(R.2) \quad \frac{d\xi}{rk} \bigg|_{\gamma=0} = \left[ \frac{r-\rho}{2r} + \frac{1-\alpha}{\alpha} - \frac{1-\alpha}{2} \frac{\rho}{r} \right] \frac{dx_i}{x} + \left[ \frac{3}{4} - \frac{r-\rho}{4r(1-\alpha)(1-\tau)} - \frac{k_i^j}{k} \right] d\tau^i
\]
Or by using the definition of \( \rho \)
\[
(R.3) \quad \frac{d\xi}{rk} \bigg|_{\gamma=0} = \left[ \frac{\tau}{2} + \frac{1-\alpha}{\alpha} - \frac{(1-\alpha)(1-\tau)}{2} \right] \frac{dx_i}{x} + \left[ \frac{3}{4} - \frac{\tau}{4(1-\alpha)(1-\tau)} - \frac{k_i^j}{k} \right] d\tau^i
\]
Rearranging gives (A.17).

**Going from (A.28) from (23)**

Writing out the terms in (A.28) gives

$$0 = k_i^j - k_j^i - k_i^j k_j^i (k_i^j - k_j^i) + 3k_i^j - k_j^i - s(k_i^j + k_j^i)$$

or

$$0 = (k_i^j)^2/k_i^j + k_j^i - s(k_i^j + k_j^i)$$

or by premultiplying by $k_i^j$ we have

$$0 = (k_i^j)^2 + (1-s)k_i^j k_j^i - s(k_i^j)^2$$

which is a quadratic equation with the only positive root $k_i^j = sk_i^j$.

**Going from (A.30) from (A.31)**

Equation (A.30) may be written as

$$\left[ [1-s] \frac{(1-a)(1-\tau)}{2} + \frac{2-a}{2a} (1-a + a \tau) \right] \frac{rk}{x} - \frac{1}{\varepsilon} g^\sigma D'(\bar{\chi}) = 0$$

or

$$\left[ [1-s](1-a)(1-\tau) + \frac{2-a}{a} (1-a + a \tau) \right] \frac{rk}{x} - \frac{2}{\varepsilon} g^\sigma D'(\bar{\chi}) = 0$$

or

$$\left[ (1-a)(1-\tau) - s(1-a)(1-\tau) + \frac{2-a}{a} - (2-a)(1-\tau) \right] \frac{rk}{x} - \frac{2}{\varepsilon} g^\sigma D'(\bar{\chi}) = 0$$

or

$$\left[ -(1-\tau) - s(1-a)(1-\tau) + \frac{2-a}{a} \right] \frac{rk}{x} - \frac{2}{\varepsilon} g^\sigma D'(\bar{\chi}) = 0$$

from which (A.31) follows.

**Going from (C.3) from (C.5)**

Inserting (C.4) into (C.3) gives
\[ \frac{d\tau^x}{\tau^x} \left[ 1 + \frac{1 - \alpha}{\alpha} \frac{1 + \sigma \beta \rho}{1 + \beta \rho} \right] = \frac{d\beta}{\beta} \frac{1 + \sigma \beta \rho}{1 + \beta \rho} \]

or \[ \frac{d\tau^x}{\tau^x} \left[ 1 + \beta \rho + \frac{1}{\alpha} \left( 1 + \sigma \beta \rho \right) \right] = \frac{d\beta}{\beta} \left( 1 + \sigma \beta \rho \right) \]

or \[ \frac{d\tau^x}{\tau^x} \left[ \frac{1}{\alpha} + \left( 1 + \frac{1 - \alpha}{\alpha} \sigma \right) \beta \rho \right] = \frac{d\beta}{\beta} \left( 1 + \sigma \beta \rho \right) \]

which gives (C.5).