DYNAMIC TAX EARMARKING AND ENDOWENOUS FISCAL STRUCTURE *

Thomas I. Renström
University of Durham, and CEPR
Revised December 2001

The purpose of the paper is to analyse how earmarking of tax revenue affects the politico-economic equilibrium, in an overlapping generations economy. Two taxes (capital, and labour) and two types of transfers (income allowance and social security) and public goods provision are analysed. Predetermined fractions of the different taxes are used for public goods provision, which affects the utility of both generations, and the remaining fractions for social security to the old generation. Individuals within each generation differ in abilities. This intragenerational difference gives rise to different preferences over the tax rates and the resulting level of public goods provision and level of social security. The fiscal policy decision is endogenised through majority voting on representatives. Among the results are: in an economy without the labour income tax the earmarking does not affect the tax rate on capital income, nor the level of public goods. The more the taxes are earmarked to social security (as opposed to public goods) the greater is the wage tax, the smaller is the steady-state capital stock, and the greater is the social security. The lower the skill of the decisive individual the greater are the income allowance, the wage tax and the social security.

Keywords: Tax earmarking, redistributive taxation, endogenous taxation, overlapping generations, social security.

JEL classification: D70, E21, E62, H20, H55.

Correspondence to: T.I. Renström, Department of Economics and Finance, University of Durham, 23-26 Old Elvet, Durham DH1 3HY, UK. Fax: +44-(0)191-334 6341.
Email t.i.renstrom@durham.ac.uk

*I would like to thank Tim Besley, Eckhard Janeba, and Michael Smart for helpful comments. Thanks are also due to participants at the ISPE-ITIC-IFS Conference, Oxford, and at the Department of Economics Workshop, Birmingham.
1. INTRODUCTION

Tax earmarking, would be expected to affect the decision upon tax rates and public expenditure only to the extent that either (a) the earmarking rules are decided upon by different agents than the agents deciding upon fiscal policy and expenditure, or (b) the earmarking rules are decided upon at a different point in time than the fiscal policy and expenditure decision. If (a) or (b) does not hold, earmarking would not be expected to change anything. For earmarking to affect the fiscal policy and expenditure decisions, it has to be a binding constraint. Whether such constraint is desirable or not would depend on the degree of time-inconsistency in policy making. In both case (a) and (b) the earmarking rules act as a partial precommitment mechanism and as such may be welfare improving. See Marsiliani and Renström (2000). There may be reasons why full commitment to future taxes could be undesirable, e.g. in the presence of uncertainty.

The purpose of this paper is to analyse the effect on fiscal decisions of different earmarking rules, i.e. the effect of having different tax revenues earmarked to different expenditure decisions. We shall focus on earmarking of benefits and public goods. Since benefits usually are of two types: affecting either the young or affecting the old, it seems most appropriate to adopt an overlapping generations framework.

We will focus on two types of benefits: lump-sum allowance to the young and social security in the form of pensions to the old. We will also incorporate public goods and

---

1 We should think of tax earmarking as "dedicating specific revenues to the financing of specific public services," see Buchanan (1963) pp. 457-458.

2 We may view case (a) as strategic delegation, i.e. the fiscal authorities delegate the decision on earmarking rules to agents with different preferences than themselves.

3 Dye and McGuire (1992) empirically investigated the impact of earmarking on the level and composition of government expenditure in the US. They found that increases in earmarked revenue cause no or little change in government expenditure. If earmarking is specified as a share of expenditure there was either no effect of earmarking or sometimes earmarking caused a reduction in expenditure.
consumed equally by the young and the old. These expenditures, which seem to be the most important in most democracies, are funded through taxes on capital income (affecting only the old), and on labour income (affecting only the young).

The view on earmarking rules follows Marsiliani and Renström (2000). A fraction of each source of tax revenue will be designated to each type public expenditure (social security and public goods). We will treat the lump-sum transfer to the young as a part of the labour tax system (i.e. the negative of the lump-sum transfer is a source of labour-tax revenue), allocated among social security and public good according to the earmarking rules. Thus we will have two earmarking rules, one for each tax.

The paper is related to the political-economy literature on taxation, Meltzer and Richard (1981), Persson and Tabellini (1994a,b) and Renström (1996). Because we have an overlapping-generations economy, where two generations exist simultaneously, and because we allow for several tax instruments we follow more closely Renström (1996).

The paper is structured as follows. In section 2 the overlapping-generations economy is introduced and the assumptions are formalised, and in section 3 the economic equilibrium is solved for. In section 4 individuals’ preferences over taxation are characterised. Section 5 solves for the politico-economic equilibrium, given the earmarking rules. The equilibrium as a function of earmarking rules is characterised. Section 6 concludes the paper.

---

4 There are two other views on earmarking in the literature. Pirttilä (1998) argues that earmarking revenues from an environmental tax to compensate those who have to bear most of the tax burden may be desirable. Brett and Keen (1998) view earmarking as a way to prevent politicians with discretionary power over the use of the revenues from deviating from the original policy proposal. See also the discussion in Marsiliani and Renström (2000).

2. THE ECONOMY

Individuals live for two periods, consuming both as young and as old, but work only when young. They have preferences over period-one consumption, period-one labour supply, period-two consumption, and period-one and period-two provision of public goods.

Individuals within each age group differ in productivity (skill). They are indexed by $i$ and characterised by their productivity parameter $\gamma_i$. The distribution of individuals, $F(i)$, is continuous and is assumed to be stationary over time, and the population grows at a constant rate $n$.

In period one individual $i$ born at $t$ (with ability $\gamma_i$) receives a lump-sum transfer $T_i$ (possibly negative), supplies labour $l^it$ on the market and consume $c^it$ units of the only consumption good. She is paid $\omega_t$ per efficiency unit of supplied labour, i.e. in proportion to $\gamma^it$, and she saves $k^{i+1}t$ for the next period. Let $\tau^l_t$ and $\tau^k_t$ denote the wage-income tax rate and the capital-income tax rate respectively. It is convenient to define the gross return on capital as $R_t \equiv (1+r_t)$ and the after-tax prices as $P_t \equiv 1 + (1-\tau^l_t)r_t$ and $\omega_t \equiv (1-\tau^k_t)\omega_t$. In period two she receives after-tax return, $P_{i+1}$, on her savings and social security $s_{i+1} = s_{i+1} \omega_t \gamma^it$, (i.e. proportional to past wage earnings) which are used for consumption $c_{i+1}$. The period-one and period-two per-capita consumption of the public good (equal for all individuals) are denoted $g_t$ and $g_{i+1}$ respectively.

2.1 Assumptions

The basic assumptions follow Renström (1996) with some modifications: allowing for social security, transfers, and earmarking of tax revenue.
A1 Individual Preferences

For analytical tractability the utility function is assumed to be of the form

$$U^H(\cdot) = c^H_t + \eta \ln(L - l^H_t) + \beta \ln c^H_{t-1} + \epsilon \ln g_t + \beta \epsilon \ln g_{t-1}$$

(1)

where $l^H_t \geq 0$, and the parameters $\beta$, $\eta$ and $\epsilon$ are strictly positive. Any individual is assumed to choose her quantities of consumption, labour supply, and savings so as to maximise (1) subject to her budget constraints.

A2 Individuals’ Constraints

The individual budget constraints are

$$c^H_t + k^H_{t+1} - \omega \gamma l^H_t + T_t$$

(2)

$$c^H_t = p_{t+1} k^H_{t+1} + s_{t+1} \omega \gamma l^H_t$$

(3)

A3 Production

A large number of firms are operating with a linear technology. Therefore aggregate production, $y_t$, can be calculated as if there was a representative firm employing the aggregate quantities of capital and labour, respectively defined as $\bar{K}_t = \int k^H_i dF(i)$ and $\bar{L}_t = \int l^H_i dF(i)$. Thus

$$y_t = f(\bar{K}_t, \bar{L}_t) = r \bar{K}_t + w \bar{L}_t$$

(4)

where the marginal products $r$ and $w$ are independent of economic activity and time.

A4 Government’s Constraint

The tax receipts at time $t$ is fully used for provision of the public good $G_t$ and social security $S_t$. A fraction, $\alpha^j_t$, of the tax receipts from tax $j = \{k, l\}$ is used for social security in the form
of pensions to the old generation, and the fraction $1 - \alpha_t^i$ is used for public goods provision.

$$S_t = \alpha_t^i (R_t - P_t) K_t + \alpha_t^i [(w_t - \omega_t) \bar{T}_t - T_t] N_t$$

(5)

$$G_t = (1 - \alpha_t^i) (R_t - P_t) K_t + (1 - \alpha_t^j) [(w_t - \omega_t) \bar{T}_t - T_t] N_t$$

(6)

Also we assume $\alpha_t^i < 1$, i.e. the wage tax cannot be fully earmarked to social security.

### A5 Altruism

It is assumed that any individual cares about the (utilitarian) welfare of the old generation and of the young generation. The objective of individual $i$ when stating preferences over policy is private utility [equation (1)] plus

$$\nu W_t^i + \delta W_t^{i-1} = \nu \int U^i dF(i) + \delta \int U^{i-1} dF(i)$$

(7)

where $\nu$ and $\delta$ reflect the degree of altruism. To allow for the possibility of young and old having different preferences, an old person’s preference parameters are denoted by tilde ($\nu, \delta$), and we assume $\delta(1 + \nu) \geq \delta(1 + \bar{\nu})$.

### A6 Population growth

Population growths at a constant rate $n$

$$N_t = (1 + n) N_{t-1}$$

(8)

### A7 Representative democracy

The tax rates, $\tau_t^i$, $\tau_t^k$, and the lump-sum transfer, $T_t$, and consequently the spending decisions are determined by a majority elected representative at time $t$. 

5
The earmarking rules, A4, imply that a fraction of each type of tax revenue has to be devoted to each of the types of public expenditure (i.e. to social security in the form of pensions and to public goods provision). The lump-sum transfer is not treated as expenditure, but as a part of a labour-income-tax system. In this way we allow for a piecewise linear labour income tax.

The altruism is specified in such a way that the young generation care about the old when making the fiscal decision (not when making private consumption decisions since each individual is "small"). If this was not the case the young would always confiscate the capital stock and give no social security in the form of pensions. Furthermore, the last condition in A5 assumes that the old care "enough" about their own generation relative to the young. This condition is a sufficient condition for an old not being majority elected.

The assumption of quasi-linear utility, with period-one consumption entering linearly, implies that all individuals plan to have the same consumption as old (regardless of their skill). Therefore in equilibrium taxation will have no redistributive effect among the old. This enables us to more clearly focus on the distributional conflict within the young generation only.

3 ECONOMIC EQUILIBRIUM

In this section the individual and aggregate economic behaviour are solved for, given any arbitrary sequences of tax rates, public expenditure and social security. By profit maximisation the before-tax prices (the interest rate and the average wage rate) are given by \( r \) and \( w \).

---

7 It turns out that population growth makes the young in majority.
3.1 Individual Economic Behaviour

Maximisation of (1) subject to (2)-(3) gives the individuals’ decision rules

\[ c_t^i = \left(1 + \frac{S_{t+1}}{P_{t+1}}\right) \omega_t \gamma^t L - (\beta + \eta) + T_t \quad (9) \]

\[ l_t^i = L - \frac{\eta}{\omega_t \gamma^t} \frac{P_{t+1}}{P_{t+1} + S_{t+1}} \quad (10) \]

\[ k_{t+1}^i = \beta - \eta \frac{S_{t+1}}{P_{t+1} + S_{t+1}} - \frac{S_{t+1}}{P_{t+1}} \omega_t \gamma^t L \quad (11) \]

\[ c_{t+1}^i = \beta P_{t+1} \quad (12) \]

Equations (9)-(12) form the complete solution to the individual optimisation problem. A direct property of quasi-linear preferences is that all income effect is carried over to the argument that enters linearly, in this case the first-period consumption. An increase in lump-sum allowance or in the present value of future social security therefore makes the individual consume more in the first period. This is why the lump-sum allowance only appears in equation (9).

3.2 Aggregate Economic Behaviour

Because of the OLG structure of the economy it is not possible to have an aggregation theorem that gives a representative individual result. However the aggregate economic behaviour is easily described anyway. Aggregate consumption at date \( t \) consists of total consumption of the young generation and total consumption of the old (born at \( t-1 \), i.e. \( C_t = N_t c_t^y dF(i) + N_{t-1} c_t^{y-1} dF(i) \). Alternatively we may define (young) population average as \( \bar{c}_t = \frac{C_t}{N_t} = \int c_t^y dF(i) + (1+n)^{-1} \int c_t^{y-1} dF(i) \). Then by using (9) and (2) at date \( t \), we have
\[ \bar{c}_t = \left(1 + \frac{s_{t-1}}{P_{t-1}}\right)\omega_t L - \beta - \eta + T_t + \frac{\varphi_t k_t + s_t \omega_{t-1} \bar{l}_{t-1}}{1 + n} \]  

Similarly for the per capita (young generation) labour supply use (10)

\[ \bar{l}_t = L - \frac{\eta}{\omega_t \frac{P_{t+1}}{P_{t+1} + s_{t+1}}} \]  

Finally the aggregate capital stock is obtained by integrating (11) (shifted back to \( t \))

\[ \bar{k}_t = \beta + \eta \frac{s_t}{P_t + s_t} - \frac{s_t \omega_{t-1} L}{P_t} \]  

Dividing through (5) and (6) by \( N_t \) the per capita social security and public goods are respectively

\[ \frac{s_t \omega_{t-1} \bar{l}_{t-1}}{1 + n} = \alpha_t [R_t - P_t] \frac{\bar{k}_t}{1 + n} + \alpha_t [w_t - \omega_t] \bar{l}_t - T_t \]  

\[ g_t = (1 - \alpha_t) [R_t - P_t] \frac{\bar{k}_t}{1 + n} + (1 - \alpha_t) [w_t - \omega_t] \bar{l}_t - T_t \]

### 4 PREFERENCES OVER TAXATION

Here we shall solve for the policy preferred by a potentially elected candidate. It will turn out that this individual belongs to the young generation, therefore we will in this section only solve for a young person’s policy preferences and leaving the old to the appendix. In choosing her preferred values of \( \omega_t \) and \( P_t \), the individual maximises (1) with respect to policy subject to (2)-(3), and (13)-(17). The individual decision rules, (9)-(12), can be substituted into the utility function, obtaining indirect utility as a function of policy. For an old individual we have
\[ V^{t+1} = \ln(P_t k_i^t + s_i \omega_{t-1} V^{t-1}_{t-1}) + \epsilon \ln g_t \] (18)

and for a young individual we have

\[ V^t = \omega_i \gamma L \left(1 + \frac{s_{t+1}}{P_{t+1}}\right) + T_i + \eta \ln \left(\frac{1}{\omega_i P_{t+1}}\right) + \beta \ln P_{t+1} + \epsilon \ln g_t + \Delta \] (19)

where \( \Delta \) incorporates constants and public goods at \( t+1 \). The objective of an individual in stating preferred policy, taking altruism into account, is

\[ J^t = V^t + \psi W_t^t + \delta W_{t+1}^t \] (20)

for a young individual, and

\[ J^{t-1} = V^{t-1} + \psi W_t^t + \delta W_{t+1}^{t-1} \] (21)

for an old individual, where \( W_t^t \) and \( W_{t+1}^t \) are the welfare measures (utilitarian) of the young and old generation respectively

\[ W_{t+1}^t = \int \ln(P_t k_i^t + s_i) dF(i) + \epsilon \ln g_t \] (22)

\[ W_t^t = \int V^t dF(i) \] (23)

We shall ask a young individual \( i \) to maximise (20) subject to (13)-(14), (16)-(17), (19), (22)-(23). We examine each of the first-order conditions for \( P_t, T_i \) and \( \omega_i \) below, in order to see exactly how the earmarking potentially affects the fiscal decisions.

4.1 The capital-income tax

The first-order condition with respect to \( P_t \) is
The first term in (24) is the increase in welfare of the old generation due to an increase in their after tax return on savings. The second term is the change in utility of the individual and in welfare of young and old generation, when the level of the public good changes as a result of changing the after-tax return on savings. This term is negative as long as some of the tax revenue from capital income is earmarked for public goods provision. The third term is the reduction in welfare of the old when the social security rate is decreased due to a reduction in the capital tax (\(=\)increase in \(P_t\)). This effect is present as long as \(\alpha_t^k > 0\), i.e. as long as some capital-tax revenue is earmarked for social security.

Only if the capital tax is fully earmarked to social security, i.e. \(\alpha_t^k = 1\), a change in the capital tax does not change the welfare of the old (at \(t\) the capital tax is not distortionary and acts simply as lump-sum tax fully returned through \(s_t\) if \(\alpha_t^k = 1\)).

We may further rearrange (24) to obtain, at the optimum,

\[
\frac{\delta}{\partial P_t} \left( \frac{\delta k_t}{P_t k_t + s_t \omega_{t-1} l_{t-1}} \right) - (1 + v + \delta) \frac{\bar{e}}{g_t} \frac{k_t}{1 + n} (1 - \alpha_t^k) - \frac{\delta \bar{k}_t \alpha_t^k}{P_t k_t + s_t \omega_{t-1} l_{t-1}} = 0
\]  

(24)

The left-hand side of (25) is the marginal welfare of old consumption, and the right-hand side is the marginal welfare of public goods times the marginal rate of transforming one unit of the public good into one unit of old consumption, which is \((1 + n)^{-1}\). Thus, this is the Samuelson rule when looking at old consumption only. The reason for this is that the capital tax is a lump-sum tax on the old generation, since \(\bar{k}_t\) is inelastic at \(t\). We also note that the desirability of the Samuelson rule is unaffected by the earmarking rule for the capital tax.

Next, if \(P_t\) was rationally expected at \(t-1\), then at the optimum we have
implying that if all young prefer the same level of public goods they would also all agree upon the capital tax.

We should also notice that if there was no altruism, $\delta=0$, the first-order variation (25) is always negative and it is optimal to set $P_t$ as small as possible. This would involve confiscation and no savings would be made in equilibrium.

4.2 The lump-sum allowance/tax

The first-order condition with respect to $T_t$ is

$$\frac{\partial J_t}{\partial T_t} = 1 + \nu - (1 + \nu + \delta) \frac{e}{\gamma_t} (1 - \alpha_0) - \frac{\delta (1 - n)}{P_t k_t + s_t \omega_{t-1} \bar{l}_{t-1}} \alpha_t = 0$$  \hspace{1cm} (27)

The first term of equation (27), $1+\nu$, is the marginal increase in utility of the individual plus the marginal increase in welfare of the young generation due to an increase in the lump-sum benefit. The second term is the change in utility of individual and in welfare of the young and old generations through the level of public goods provision. This term is negative as long as some of the labour tax revenue is earmarked for public goods provision (the benefit is then taken out of the public goods budget). The last term of equation (27) is the marginal decrease in welfare of the old generation of increasing the lump-sum transfer to the young. This term is positive as long as some of the labour-tax is earmarked to social security.

Next, combine (27) with (25) and rearrange to obtain

$$1 + \nu = (1 + \nu + \delta) \frac{e}{\gamma_t}$$  \hspace{1cm} (28)

The left-hand side is the marginal welfare of young consumption and the right-hand side is
the marginal welfare of public goods times the marginal rate of transforming one unit of the public good into a unit of young consumption. Since both are per capita quantities of the young generation the marginal rate of transformation is unity. Thus, this is the Samuelson rule in terms of young consumption.

Combining (28) with (26) gives

$$P_t = \frac{\delta}{\beta} \frac{1+n}{1+v}$$  \hspace{1cm} (29)

We see that when the lump-sum measures (the capital tax and the lump-sum transfer to the young) are optimally chosen by a young individual (and they were expected to be so at \(t-1\)), all young agree upon the level of public goods provision and on the capital tax.

We should also notice that the lump-sum transfer decision is still well defined even if \(\delta=0\) [by equation (27)] as long as \(\frac{\partial g}{\partial T} < 0\), i.e. as long as some labour-tax revenue is earmarked \((1-\alpha_t) > 0\). Equations (28)-(29) would not be valid if \(\delta=0\), since we could not use (25) as an intermediate step.

4.3 The labour tax

In individuals’ utilities and in the function for aggregate labour supply the future social-security rate and the future capital-income tax enter, therefore the individuals have to figure out what the future government is going to do. When the present government is choosing the labour tax it must know how it affects the future government’s decision (which in turn is a function of government’s decision two generations ahead, and so on). The future government’s optimal social security rate will be a function of after-tax wage chosen by the present government, \(s_{t+1} = s(\omega_t)\). The future capital income tax, if a young representative is elected next period, is unaffected by the after-tax wage chosen by the present government,
by equation (29). We may then treat aggregate labour supply as a function of the present after-tax wage and the expected future social security, in turn as a function of the present after-tax wage, \( \bar{l}_t = \bar{l}(\omega_t, s(\omega_t)) \equiv \phi(\omega_t) \). We refer to \( \phi'(\omega_t) \) as \( d\bar{l}/d\omega_t \), i.e. the total derivative of labour supply with respect to present after-tax wage. Next, we may rewrite utility (19) in terms of the aggregate labour supply, by substituting for the terms involving future social security, using equation (14). Then we have

\[
V^{it} = \frac{\eta^i L}{L - \bar{l}_t} + T_t + \eta \ln(L - \bar{l}_t) + \beta \ln P_{t+1} + \epsilon \ln g_t + \Delta \tag{30}
\]

The first-order condition with respect to \( \omega_t \) is

\[
\frac{\partial J^{it}}{\partial \omega_t} = \left[ \frac{(\gamma^i + v) \eta L}{(L - \bar{l}_t)^2} - \frac{(1 + v) \eta}{L - \bar{l}_t} \right] \frac{d\bar{l}_t}{d\omega_t} + \left[ 1 + v + \delta \right] \frac{c}{g_t} \left[ (w - \omega_t) \frac{d\bar{l}_t}{d\omega_t} - \bar{l}_t \right] (1 - \alpha^i_t) \\
+ \frac{\delta (1 + n)}{P^{k_t, s_t, \omega_t, \bar{l}_t} \bar{l}_{t-1}} \left[ (w - \omega_t) \frac{d\bar{l}_t}{d\omega_t} - \bar{l}_t \right] \alpha^i_t = 0 \tag{31}
\]

Using (27) (the optimal lump-sum transfer decision) this simplifies to

\[
\frac{\gamma^i + v}{1 + v} \eta L - \eta (L - \bar{l}_t) + (w - \omega_t) (L - \bar{l}_t)^2 = \bar{l}_t (L - \bar{l}_t)^2 \left[ \frac{d\bar{l}_t}{d\omega_t} \right] \tag{32}
\]

Next in order to find \( d\bar{l}(\omega_t, s(\omega_t))/d\omega_t \) we must know \( s'(\omega_t) \).

**Lemma 1** Assume A1-A6, then if a young person is majority elected at all times then in politico-economic equilibrium the following is true
That is, a percentage increase in the after-tax wage today will cause the future government to decrease the social security rate more than one percent.

Since

\[
\frac{d s_{t+1}}{d \omega_t} \frac{\omega_t}{s_{t+1}} = \frac{-L}{L - \bar{L}_t} \frac{P_{t+1}}{P_{t+1} + s_{t+1}} < -1 \tag{33}
\]

we have, by using Lemma 1 to substitute for \(ds(\omega_t)/d\omega_t\),

\[
\frac{d \bar{l}_t}{d \omega_t} = \frac{\partial \bar{l}_t}{\partial \omega_t} + \frac{\partial \bar{l}_t}{\partial s_{t+1}} \frac{d s_{t+1}}{d \omega_t} = \frac{L - \bar{l}_t}{\omega_t} + \frac{L - \bar{l}_t}{P_{t+1} + s_{t+1}} \frac{d s_{t+1}}{d \omega_t} \tag{34}
\]

Then, combining (32) and (35) gives the optimal aggregate labour supply desired by a decisive individual.

\[
(L - \bar{l}_t)^2 - \frac{\eta}{w} (L - \bar{l}_t) - \frac{\eta L}{w} \frac{1 - \gamma^t}{1 + \gamma^t} = 0 \tag{36}
\]

The decisive individual will choose the after tax wage in order to implement (36). We see that the aggregate labour supply (the positive root to (36)) is increasing in the skill level of the decisive individual. The reason for this is that an individual endowed with a low skill level would like to use the tax system in order to redistribute toward herself. This redistributive tax reduces the aggregate labour supply. An individual with high skill level would like to subsidise labour, which affect the aggregate labour supply positively. We will give a more
exact interpretation of the labour tax system later in section 5.

5 POLITICO-ECONOMIC EQUILIBRIUM

In this section we will solve for the endogenous fiscal policy. It will be the policy preferred by the majority elected representative. First we will characterise the individuals’ preferences over choice of policy maker. Since individuals differ in only one dimension, in skills, it amounts to choosing a skill level of the policy maker.

It will be established that individuals’ preferences over the skill level of the policy maker are single peaked, with the most preferred policy maker coinciding with own skill. Next, the politico-economic equilibrium will be characterised, and in particular how the earmarking rules affect taxes and expenditure. It will be shown that the earmarking rules do not affect the capital tax, or the level of public goods. Earmarking does affect the level of social security, the labour-income tax, the lump-sum allowance, and the steady-state levels of capital.

We begin by establish

Lemma 2 Assume A1-A6, then an old individual cannot be majority elected.

Proof: See the appendix.

Next, we have

---

8 The way in which we approach the problem should not be confused with the framework of Osborne and Slivinski (1996) and Besley and Coate (1997). In their framework there is a cost for candidates running for election. This cost reduces the number of candidates entering. In this paper there is no cost of entering and we may think of all individuals being candidates. We will have a median-voter equilibrium.
Lemma 3  Assume A1-A6, then any individual’s preferences over representatives are single peaked.

Proof: The aggregate labour supply, equation (14), may be written as

$$\frac{\bar{I}_t}{L-I_t} = \frac{\omega_t \bar{I}_t}{\eta} + \frac{\omega_t \bar{I}_t \bar{I}_{t+1}}{\eta P_{t+1}} = \frac{\omega_t \bar{I}_t}{\eta} + \frac{1+nH_{t+1}}{P_{t+1}}$$ (37)

where

$$H_{t+1} = \frac{(\alpha_{t+1} - \alpha_t) (R_{t+1} - P_{t+1}) \beta P_{t+1}/(1+n) + \alpha_t P_{t+1} g_{t+1}}{(1-H_t)P_{t+1} + (\alpha_{t+1} - \alpha_t)R_{t+1}}$$ (38)

The last equality of (37) follows from (59) in the appendix.

Rearrange (37)

$$\omega_t = \frac{\eta}{L-I_t} - \frac{1+nH_{t+1}}{I_t P_{t+1}}$$ (39)

This implies that the lump-sum transfer $T_t$ may be rewritten in terms of aggregate labour supply only (by substituting for the after-tax wage, equation (39)). Use equation (56) in the appendix, then we have

$$T_t = \frac{1-\alpha}{1-\alpha_t} (R_t - P_t) \frac{k_t}{1+n} + w_t \bar{I}_t - \frac{\eta \bar{I}_t}{L-I_t} + (1+n) \frac{H_{t+1}}{P_{t+1}} - \frac{e}{1-\alpha_t} \frac{1+n+\delta}{1+n}$$ (40)

We may then write young indirect utility, equation (30) as
The preferences over candidates are therefore characterised by

\[ V^u = \frac{\eta \gamma^i L}{L - \bar{L}_t} + \frac{1 - \alpha^i}{1 - \alpha^i} (R_t - P_t) \bar{k}_t \frac{1}{1 + n} + w_t \bar{L}_t - \frac{\eta \bar{L}_t}{L - \bar{L}_t} + (1 + n) \frac{H_{t+1}}{P_{t+1}} \]

\[ - \frac{\epsilon}{1 - \alpha^i} \frac{1 + \nu + \delta}{1 + \nu} + \eta \ln (L - \bar{L}_t) + \beta \ln P_{t+1} + \epsilon \ln g_t + \Delta \]

(41)

for a young voter, and by

\[ J^{u^*} = \frac{\eta (\gamma^i - \nu) L}{L - \bar{L}_t} + (1 + \nu) w_t \bar{L}_t - \frac{(1 + \nu) \eta \bar{L}_t}{L - \bar{L}_t} + (1 + \nu) \eta \ln (L - \bar{L}_t) \]

(42)

for an old voter.

Differentiating young voters’ utilities with respect to candidate skill gives

\[ \frac{\partial J^{u^*}}{\partial \gamma^d} = \left[ \frac{\eta (\gamma^i + \nu) L}{(L - \bar{L}_t)^2} + (1 + \nu) w_t \bar{L}_t - 2 \frac{(1 + \nu) \eta \bar{L}_t}{(L - \bar{L}_t)^2} \right] \frac{\partial \bar{L}_t}{\partial \gamma^d} \]

(44)

or by rearranging and substituting for the optimal policy of the elected individual (equation (36)) we have

\[ \frac{\partial J^{u^*}}{\partial \gamma^d} = \frac{\eta L}{(L - \bar{L}_t)^2} (\gamma^i - \gamma^d) \frac{\partial \bar{L}_t}{\partial \gamma^d} \]

(45)

Similarly, differentiating old voter utility (equation (43)) with respect to candidate skill gives

\[ \frac{\partial J^{u^*-1}}{\partial \gamma^d} = \left[ \frac{\eta \bar{v} L}{(L - \bar{L}_t)^2} + \bar{v} w_t \bar{L}_t - 2 \frac{\bar{v} \eta \bar{L}_t}{(L - \bar{L}_t)^2} \right] \frac{\partial \bar{L}_t}{\partial \gamma^d} \]

(46)

or by rearranging and substituting for the optimal policy of the elected individual
Equation (45) is negative for $\gamma_i < \gamma_d$ and positive for $\gamma_i > \gamma_d$, and equation (47) is negative for $1 < \gamma_i$ and positive for $1 > \gamma_i$, implying that individual utilities are decreasing everywhere outside the most preferred candidate. QED

We should note that all old prefer a candidate with $\gamma_i = 1$ (i.e. with mean skill). Therefore a considerable skewness in skill distribution is required to obtain a decisive individual with less skill than the mean. The intuition for this result is that the old care only about social welfare, since in equilibrium there is no redistribution among the old.

In what follows we shall apply the median-voter theorem, (see e.g. Mueller (1989)). The median-voter theorem just tells that the median position cannot loose under majority rule. It does not say that if individuals cast their votes simultaneously that the median position will be the outcome. If individuals cast their votes simultaneously there may in fact be multiple voting equilibria (see Besley and Coate (1997)). We shall however take the median position as our political equilibrium.\(^9\)

Applying the median-voter theorem, (see e.g. Mueller (1989)), the candidate preferred by the median voter cannot loose under majority rule and we have

\[
\frac{\partial J_i^{\gamma^d_I}}{\partial \gamma^d} - \frac{\nu \eta L}{(L - I_i)^2} \frac{1 - \gamma^d}{1 + v} \frac{\partial I_i}{\partial \gamma^d}
\]  

\[(47)\]

\(^9\) A procedure that works is an election with only two candidates at a time, however, with a large number of individuals this would be cumbersome. We could also have assumed that there are only two skill types in the economy. Then the voting is trivial.
Theorem 1 Assume A1-A7, then the median-voter equilibrium is characterised by

\[ g_t^* = e\left(1 + \frac{\delta}{1+\nu}\right) \quad (48) \]
\[ \bar{l}_t^* \geq \frac{L - \frac{n}{w}}{\gamma_d} \implies \gamma_d \geq 1 \quad (49) \]
\[ \tilde{k}^*_t = K(a^k, a^l, \gamma_d) \quad (50) \]
\[ P_t^* = \frac{\delta(1+n)}{\beta(1+\nu)} \quad (51) \]
\[ T_t^* = T(a^k, a^l, \gamma_d) \quad (52) \]
\[ \tau_t^i = \tau_i(a^k, a^l, \gamma_d) \quad (53) \]
\[ s_t^* = s(a^k, a^l, \gamma_d) \quad (54) \]
\[ s_t^i \omega_{t-1} \tilde{l}_{t-1} = S(a^k, a^l, \gamma_d) \quad (55) \]

where the signs below the arguments are the signs of the partial derivatives.

Proof: See the appendix.

We see that all quantities are always at their steady-state values. The per-capita level of public goods is independent of the identity of the majority elected representative, because all young agree upon its level. The level of public goods obeys the Samuelson rule (see sections 4.1 and 4.2) and is also independent of the earmarking rules. Not surprisingly the level is larger the stronger the preferences for public goods are (i.e. larger $\varepsilon$).

The level is also larger the more a young cares about the old (i.e. larger $\delta$), the less a young cares about the young generation (i.e. smaller $\nu$). The reason is that it is less costly to transform public goods into old consumption than into young consumption, because of the population growth.

The capital income tax is smaller (i.e. $P_t$ larger) the more a young cares about the old (i.e. larger $\delta$), the less a young cares about the young generation (i.e. smaller $\nu$), the larger the population growth is, and the larger the rate of time preference is (i.e. smaller $\beta$). The capital-income tax is also independent of the earmarking rules.
To explain (49) we concentrate first on the case when the skill of the decisive individual is equal to the average skill. Then aggregate labour supply, $\bar{l}^*$, equals $L-\eta/w$. By inspection of equation (14) we see that this is exactly the aggregate labour supply in a situation when the labour income tax is zero and when there is no social security, i.e. there are no distortions of the labour supply decision. If it is expected that the future government chooses to give social security, the present government chooses to tax labour today so as to exactly offset the distortion implied by the future social security. Furthermore, by inspection of (40), in this case the transfer given to the young is independent of labour supply and wage, implying that there is no redistribution among the young.

If the skill of the decisive individual is lower than the mean $\bar{l}^*$ is smaller than $L-\eta/w$, and according to (14) the aggregate labour supply decision is distorted, and according to (40) the transfer is positively related to the aggregate wage bill, implying that there is redistribution among the young from high skilled to low skilled. If the skill of the decisive individual is greater than the mean the relations are reversed.

The result regarding the redistribution among the young is analogous to the results of many other political-economy models with purely redistributive taxation. In the models by Meltzer and Richard (1981), and Persson and Tabellini (1994a) the redistributive tax rate is zero if the skill of the decisive individual equals the mean skill, and positive if lower than the mean skill, and negative if greater than the mean skill. The reason is than an individual with mean skill cannot use the tax system so as to redistribute toward herself, it is then optimal for that individual to choose the tax equal to zero. The economy by Meltzer and Richard (1981) is static, so there is no intertemporal dimension. In the Persson and Tabellini (1994a) economy individuals live for two periods but two generations never overlap. So the present government does not have to figure out what the future is going to do, or take into account
what the past government did. In Renström (1996) generations overlap and the wage tax at any point in time will depend on what the past government choose it to be. This intergenerational interaction may cause the labour tax to be non-zero, even if the decisive individual has the mean skill. Accordingly, in the economy in our paper the wage tax today will depend on what the future government is going to do, and consequently the wage tax may not be zero even if the skill of the decisive individual equals the mean. We should also notice that the earmarking rules do not affect the size of the redistribution among the young. The reason for this is that the rules do not restrict the labour tax system, since a piecewise linear labour tax can be levied.

Even though the earmarking rules do not affect the size of the redistribution among the young they do affect the taxes from and transfer to the young.

The more the capital- and labour taxes are earmarked for social security the lower is the lump-sum transfer to the young (equation (52)), and the greater is the labour tax (equation 53)). The lump-sum transfer to young is used as an instrument for transferring funds from one budget to the other. The young generation has an optimal level of public goods provision, and the lump-sum transfer is used so as to reach that level. The level of the lump-sum transfer will therefore depend on to what extent the labour tax and capital tax are earmarked for social security. The more the capital tax and the labour tax are earmarked to social security the less of these taxes can be used for public goods, and therefore the lower the transfer has to be, in order to reach the optimal level of public goods.

The total level of social security payed out at date $t$, equation (55) is independent of the labour tax and labour supply at date $t$. The reason for this is that any effect the labour supply or the labour tax has on the level of public goods spending has to be offset by a
change in the lump-sum transfer. This implies that any potential effect of labour supply and
labour tax on social security is exactly offset by the change in the lump-sum transfer. The
reason for this is that the labour tax and transfer is earmarked in the same way (i.e. with the
same $\alpha$). For this reason the identity of the decisive individual does not affect the total level
of social security (equation (55)). Since the labour tax is greater and the labour supply is
lower when the skill of the decisive individual is lower, the social security rate has to be
larger. Thus the lower the skill of the decisive individual is the larger is the social security
rate (equation (54)).

6 CONCLUSIONS

The tax earmarking affects the lump sum transfer, the labour-income tax, the social security
rate, the level of social security, and the steady-state level of the capital stock, in politico-
economic equilibrium. The more the capital-tax and labour-tax revenues are earmarked to
social security the greater are the labour-income tax, the social-security rate and the level of
social security, and the lower is the steady-state capital stock.

The skill-level of the elected representative also affect the equilibrium. The lower the
skill the greater are the lump-sum transfer, the labour-income tax, the social-security rate and
the level of social security, and the lower is the steady-state capital stock.

The tax earmarking rules do not affect the level of public goods. The reason for this
is that the government may use the lump-sum transfer and the capital tax so as to reach the
most preferred provision level.
Proof of Lemma 1

Substitute for the public-good rule (28) into (17) to obtain the transfer to young

\[ T_t = \frac{1 - \alpha_i^k}{1 - \alpha_i^l} (R_t - P_t) \frac{\bar{k}_t}{1 + n} + (w_t - \omega_t) \bar{l}_t - \frac{\epsilon}{1 - \alpha_t^l} \frac{1 + \nu + \delta}{1 + \nu} \]  

(56)

Substitute into the equation for social security

\[ \frac{s_t \omega_{t-1} \bar{l}_{t-1}}{1 - n} = \frac{\alpha_i^k - \alpha_i^l}{1 - \alpha_i^l} (R_t - P_t) \frac{\bar{k}_t}{1 + n} + \frac{\alpha_i^l}{1 - \alpha_i^l} \epsilon \frac{1 + \nu + \delta}{1 + \nu} \]  

(57)

Shift forward

\[ \frac{s_{t+1} \omega_{t} \bar{l}_{t}}{1 + n} = \frac{\alpha_i^k - \alpha_i^l}{1 - \alpha_i^l} (R_{t+1} - P_{t+1}) \frac{\bar{k}_{t+1}}{1 + n} + \frac{\alpha_i^l}{1 - \alpha_i^l} \epsilon \frac{1 + \nu + \delta}{1 + \nu} \]  

(58)

Combining (14) and (15) evaluated at \( t+1 \) gives \( \bar{k}_{t+1} = \beta - s_{t+1} \omega_{t+1} \bar{l}_{t+1} / P_{t+1} \), which substituted into (58) gives (after rearranging)

\[ \frac{s_{t+1} \omega_{t} \bar{l}_{t}}{1 + n} = \frac{(\alpha_i^k - \alpha_i^l)(R_{t+1} - P_{t+1}) \beta P_{t+1} / (1 + n) + \alpha_i^l P_{t+1} g_{t+1}}{1 - \alpha_i^k P_{t+1} + (\alpha_i^k - \alpha_i^l) R_{t+1}} = H_{t+1} \]  

(59)

Since \( P_{t+1} = \delta(1+n)(1+\nu)\beta^2 \) and \( g_{t+1} = \epsilon(1+\delta+\nu)/(1+\nu) \) in optimum, the right-hand side (= \( H_{t+1} \)) is just a constant. Differentiating (59) with respect to \( \omega_t \), we have

\[ \frac{d s_{t+1} \omega_t \bar{l}_{t}}{d \omega_t} = s_{t+1} \bar{l}_{t} + s_{t+1} \omega_t \frac{d \bar{l}_t}{d \omega_t} = 0 \]  

(60)
Next, differentiating (14)

\[ \frac{d\tilde{l}_t}{d\omega_t} = \frac{\partial \tilde{l}_i}{\partial \omega_t} + \frac{\partial \tilde{l}_i}{\partial s_{t+1}} \frac{ds_{t+1}}{d\omega_t} = \frac{L - \tilde{l}_i}{\omega_t} + \frac{L - \tilde{l}_i}{P_{t-1} + s_{t+1}} \frac{ds_{t+1}}{d\omega_t} \]  \hspace{1cm} (61)

Combining (60) and (61) gives (33). QED

**Old individual’s preferences**

Before proving that an old individual can never be majority elected, we shall prove that all old prefer no intra-generational redistribution among the young.

**Lemma 0** Assume A1-A6, if a young person is expected to be majority elected at t+1, and if an old person were to chose policy at t, then the old person prefers the same intra-generational transfer as a young person who is endowed with mean skill.

**Proof**: The first-order conditions with respect to \( T_t \) and \( \omega_t \) are

\[ \frac{\partial J_{t+1}^{u-1}}{\partial T_t} = 0 = \tilde{v} - (1 + \tilde{v} + \tilde{\delta}) \frac{c}{B_t} (1 - \alpha_t^l) \]

\[ - \left[ \frac{\omega_{t-1} \gamma_{t, t-1}^{1-1}}{P_t k_t^l + s_t \omega_{t-1} \gamma_{t, t-1}^{1-1}} + \tilde{\delta} \int \frac{\omega_{t-1} \gamma_{t, t-1}^{1-1}}{P_t k_t^l + s_t \omega_{t-1} \gamma_{t, t-1}^{1-1}} dF(t) \right] \frac{1 + n}{\omega_{t-1} \tilde{l}_{t-1}} \alpha_t^l \]  \hspace{1cm} (62)
respectively. (62) implies

\[
\left[ \frac{\omega_{t-1} \gamma I_{t-1}}{P_t k_t + s_t \omega_{t-1} \gamma I_{t-1}} \right] + \delta \int \frac{\omega_{t-1} \gamma I_{t-1}}{P_t k_t + s_t \omega_{t-1} \gamma I_{t-1}} dF(i) \left[ \frac{(1+n) \alpha_t}{\omega_{t-1} I_{t-1}} + \tilde{\nu} - (1+\tilde{\nu} + \delta) \frac{\epsilon}{g_t} (1-\alpha_t) \right]
\]

Substituting (64) into (63) gives after rearrangement

\[
\frac{\partial J^{u-1}}{\partial \omega_t} = \tilde{\nu} \left[ \frac{\eta L}{(L - \bar{L}_t)^2} - \frac{\eta}{L - \bar{L}_t} \right] \frac{\partial \bar{L}_t}{\partial \omega_t} + \tilde{\nu} \left( -\bar{L}_t + (w - \omega_t) \frac{\partial \bar{L}_t}{\partial \omega_t} \right) = 0
\]

or

\[
\left[ \frac{\eta L}{(L - \bar{L}_t)^2} - \frac{\eta}{L - \bar{L}_t} + w - \omega_t \right] \frac{\partial \bar{L}_t}{\partial \omega_t} = \bar{L}_t
\]

Using (35) and rearranging gives

\[ L - \bar{L}_t = \eta / w \]  

Which is the labour tax and transfer preferred by an individual with mean skill (equation (36)).

Proof of Lemma 2

We shall evaluate the first-order condition for $T_t$ for the old at the policy preferred by the
young. Denote the policy preferred by the young as $P_t^y$ and $g_t^y$. Since a young person is expected to be majority elected the denominators in the integrals are not individual specific, and simplify to

$$P_t^y k_t^y + s_t \omega_{t-1}^y \gamma_{t-1}^y = P_t^y k_t^y + s_t \omega_{t-1}^y \gamma_{t-1}^y = P_t^y \beta = \delta \frac{1+n}{1+v}$$  \hspace{1cm} (68)$$

The first and second equalities follow from (10)-(11), (14)-(15), and the last from (29).

Then (62) becomes

$$\frac{\partial J_{i t}^{i-1}}{ \partial T_t} \bigg|_{P_t^y, g_t^y} = \nu - (1+\nu + \delta) \frac{1-\alpha_t^l}{g_t^y} - \left( \frac{\gamma_{t-1}^l}{l_{t-1}^l} + \frac{1}{\delta} \right) \frac{1+v}{\nu} \alpha_t^l$$ \hspace{1cm} (69)$$

Using (28) to substitute for $g_t$ and rearranging gives

$$\frac{\partial J_{i t}^{i-1}}{ \partial T_t} \bigg|_{P_t^y, g_t^y} \bigg|_{P_t^y, g_t^y} = -(1+\nu + \delta) \frac{1-\alpha_t^l}{g_t^y} + \frac{1+v}{\nu} \alpha_t^l,$$

which clearly is negative for $\delta (1+\nu) \geq \nu (1+\nu)$. Thus at the optimal policy of a young individual the old would like to reduce $T_t$, i.e. would like to give less transfer. Any young would then be better off voting for a young individual with mean skill rather than voting for an old (Lemma 0). Thus an old cannot be majority elected. QED

Proof of Theorem 1

Equations (48)-(49), (51) follow from (28), (36) and (29) respectively.

First, from (59) we have

$$H_t = H(\alpha_t^l, \alpha_t^k, \eta_t^l, 0)$$ \hspace{1cm} (71)$$

where the signs are the partial derivatives. $H_t$ is also increasing (decreasing) in $n$ if $\alpha_t^l > (<)$
α_t^k. Since total social security is \( S_t = (1+n)H_t/\omega_t \bar{L}_t \), we have, by substituting for \( \omega_t \bar{L}_t \) using (39),

\[
\frac{S_t}{1+n} = \frac{H_t}{\eta \frac{\bar{L}_{t-1}}{L-\bar{L}_{t-1}} - \frac{(1+n)H_t}{\eta P_t}} \tag{72}
\]

Thus

\[
S_t = S(n, H_t, \eta, \bar{L}_{t-1}, P_t) \tag{73}
\]

Combining with (71) gives (55).

Combining (14) and (15) and using the definition of \( H_{t+1} \) gives \( k_{t+1} = \beta - H_{t+1}/P_{t+1} \).

Implies that the capital stock and \( H_{t+1} \) are negatively related. The rest follows from (71).

The properties of (53) are obtained as follows. \( \omega_t \) and \( \tau_t^l \) are negatively related. By (39) \( \omega_t \) is increasing in \( \bar{L}_t \), which in turn is increasing in the skill of the decisive individual. Thus \( \tau_t^l \) is decreasing in the skill of the decisive individual. Next, since \( \omega_t \) and \( H_{t+1} \) are negatively related, anything increasing \( H_{t+1} \) increases \( \tau_t^l \). The rest follows from (71).

To prove (52) differentiate the transfer \( T_t \) with respect to the earmarking rules. Both derivatives are negative. Finally (54) is proven by using (39) and (73). QED
REFERENCES


Persson, Torsten, and Guido Tabellini (1994a), "Is Inequality Harmful for Growth? Theory

