Decentralized Computations by Mobile Agents in Time-Varying Graphs

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Decentralized Computations by Mobile Agents in Time-Varying Graphs

joint work with

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DISTRIBUTED COMPUTING by COMPUTATIONAL ENTITIES

**OPERATE AND MOVE IN A DISCRETE SPACE**

Graph \( G = (V, E) \)

- \( V \) nodes (sites, hosts)
- \( E \) edges (links, channels)

called *agents* or *robots*
Each node has a distinct label for its links.
Each Agent

- Has computing capabilities
- Has limited storage
- Can move from node to neighboring node

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The **Agents**

Have the same behavior  (execute the same protocol)

Collectively

they perform some task  (solve a problem)
Tasks / Problems

RendezVous / Gathering

Exploration / Map Construction

Black Hole Search

Decontamination

...
Tasks / Problems

RendezVous

Gathering
Tasks / Problems

RendezVous

Gathering
- strict
Tasks / Problems

RendezVous

Gathering
- strict
- near

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### Gathering in Discrete Space

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Conference/Journal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baston, Gal</td>
<td>[<em>Naval Log. Res.</em> 91]</td>
</tr>
<tr>
<td>Alpern</td>
<td>[<em>SIAM J Cont. Optimization</em> 95]</td>
</tr>
<tr>
<td>Yu, Yung</td>
<td>[<em>ICALP</em> 96]</td>
</tr>
<tr>
<td>Alpern, Boston, Essegarer</td>
<td>[<em>J Appl. Probability</em> 99]</td>
</tr>
<tr>
<td>Howard et al</td>
<td>[<em>Operation research</em> 99]</td>
</tr>
<tr>
<td>Barrière, Flocchini, Fraigniaud, Santoro</td>
<td>[<em>SPAA</em> 03]</td>
</tr>
<tr>
<td>Dessmark, Fraigniaud, Pelc</td>
<td>[<em>ESA</em> 03]</td>
</tr>
<tr>
<td>Dobrev, Flocchini, Prencipe, Santoro</td>
<td>[<em>OPODIS</em> 03]</td>
</tr>
<tr>
<td>Kranakis, Krizac, Santoro, Sawchuk</td>
<td>[<em>ICDCS</em> 03]</td>
</tr>
<tr>
<td>Kowalski, Pelc</td>
<td>[<em>ISAAC</em> 04]</td>
</tr>
<tr>
<td>Flocchini, Kranakis, Krizac, Santoro, Sawchuk</td>
<td>[<em>LATIN</em> 04]</td>
</tr>
<tr>
<td>Dessmark, Fraigniaud, Kowalski, Pelc</td>
<td>[<em>Networks ‘06</em>]</td>
</tr>
<tr>
<td>Kranakis, Krizank, Marcou</td>
<td>[<em>LATIN</em> 06]</td>
</tr>
</tbody>
</table>

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Gathering in Discrete Space

Chalopin [Theo. Comp. Sci. ’08]
Czyzowicz, Dobrev, Kranakis, Krizanc [SOFSEM 08]
Klasing, Markou, Pelc [Theo. Comp. Sci. ’08]
Kowalski, Mailnowski [Theo. Comp. Sci. ’08]
D’Angelo, Di Stefano, Klasing, Navarra [Theo. Comp. Sci. ‘14]
Das, Luccio, Markou [ALGOSENSORS 15]
Dieudonné, Pelc, Villain [SIAM J. Comp. ’15]
Das, Luccio, Focardi, Markou, Moro, Squarcina [ICTCS 16]
Miller, Pelc [Dist. Comput. ’16]
Bouchard, Dieudonné, Ducourthial [Dist. Comput. ’16]
Dieudonné, Pelc [Algorithmica ’16]

AND MANY MORE ...
Tasks / Problems

Exploration

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## Exploration/Map Construction

<table>
<thead>
<tr>
<th>Reference</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shannon</td>
<td>JMF 51</td>
</tr>
<tr>
<td>Blum, Kozen</td>
<td>FOCS 78</td>
</tr>
<tr>
<td>Dudek, Jenkin, Milios, Wilkes</td>
<td>Robotics and Automation 91</td>
</tr>
<tr>
<td>Bender, Slonim</td>
<td>FOCS 94</td>
</tr>
<tr>
<td>Betke, Rivest, Singh</td>
<td>Machine Learning 95</td>
</tr>
<tr>
<td>Bender, Fernandez, Ron, Sahai, Vadhan</td>
<td>STOC 98</td>
</tr>
<tr>
<td>Deng, Papadimitriou</td>
<td>J. Graph Theory 99</td>
</tr>
<tr>
<td>Panaite, Pelc</td>
<td>J. Algorithms 99</td>
</tr>
<tr>
<td>Awerbuch, Betke, Rivest, Singh</td>
<td>Information and Comp. 99</td>
</tr>
<tr>
<td>Panaite, Pelc</td>
<td>Networks 00</td>
</tr>
<tr>
<td>Albers, Henzinger</td>
<td>SIAMJC 00</td>
</tr>
<tr>
<td>Duncan, Kobourov, Kumar</td>
<td>SODA 01</td>
</tr>
</tbody>
</table>

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Exploration/Map Contruction

Diks, Fraigniaud, Kranakis, Pelc [J Algorithms 02]
Fraigniaud, Ilcinkas [STACS 04]
Fraigniaud, Ilcinkas, Peer, Pelc, Peleg [MFC S04]
Das, Flocchini, Nayak, Santoro [ISAAC 06]
Gasienic, Klansing, Martin, Navarra, Zhang [SIROCCO 07]
Das, Flocchini, Kutten, Nayak, Santoro [TCS 07]

AND MANY MORE ...

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Gathering and Exploration in Discrete Space

Variety of assumptions and conditions

- Agents with/without ids
- Nodes with/without ids
- With/without orientation
- With/without tokens
- With/without faults
- A-priori knowledge of number of agents \( k \)
- A-priori knowledge of number of nodes \( n \)
- A-priori knowledge of network topology
- ...

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Gathering and Exploration in Discrete Space

SHARED ASSUMPTION:

network is static
Gathering and Exploration in Discrete Space

network is dynamic
Dynamic Networks

network is **dynamic**

topology changes continuously & unpredictably
Dynamic Networks

network is **dynamic**

topology changes continuously & unpredictably

(under the control of an adversary)

possibly disconnected
Dynamic Networks: WIRELESS MOBILE

mobile ad hoc networks (MANETS)

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Dynamic Networks: LEO SATELLITE NETWORK
Dynamic Networks: PEER-TO-PEER

OVERLAY networks

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Dynamic Networks: SOCIAL NETWORKS/WEB GRAPHS

HOTMAIL
Dynamic Network

Modeled as TIME-VARYING GRAPH


A general mathematical formalism that describes many different types of dynamic networks

A model that includes most existing models as special cases

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Time-Varying Graph

$G_t = (N, E, T, \psi, \rho, \zeta)$
Time-Varying Graph

\[ \mathcal{G} = (N, E, T, \psi, \rho, \zeta) \]
Time-Varying Graph

$G = (N, E, T, \psi, \rho, \zeta)$

$E \subseteq N \times N$
Time-Varying Graph

\[ G = (N, E, T, \psi, \rho, \zeta) \]

**lifetime of system** (contiguous time span)

\[ T \subseteq \mathcal{R} \]
Time-Varying Graph

\[ G = (N, E, T, \psi, \rho, \zeta) \]

**lifetime of system** (contiguous time span)

\[ T \subseteq \mathcal{R} \]

**Limited (finite)**
Time-Varying Graph

\[ G = (N, E, T, \psi, \rho, \zeta) \]

Lifetime of system (contiguous time span)

\[ T \subseteq \mathcal{R} \]

Unlimited (infinite)

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Time-Varying Graph

\[ G_T = (N, E, T, \psi, \rho, \zeta) \]

**lifetime of system** (contiguous time span)

\[ T \subseteq \mathbb{R} \]

Unlimited (infinite)

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Time-Varying Graph

\[ \mathcal{G} = (N, E, T, \psi, \rho, \zeta) \]

**Node presence function**
\[ \psi : N \times T \to \{0, 1\} \]
\[ \psi(x,t) = 1 \text{ iff } x \text{ is in present at time } t \]

**Edge presence function**
\[ \rho : E \times T \to \{0, 1\} \]
\[ \rho(e,t) = 1 \text{ iff } e \text{ is present at time } t \]
Time-Varying Graph

\[ \mathcal{G} = (N, E, T, \psi, \rho, \zeta) \]

**Latency (duration) function**

\[ \zeta : E \times T \rightarrow T \cup \{\perp\} \]

- \[ \zeta((x,y), t) = d \]
  - message from x to y, sent at time t, will arrive at time \( t + d \)

- \[ \zeta((x,y), t) = \perp \]
  - message from x to y, if sent at time t, will not arrive
**Time-Varying Graph: Snapshot & Footprint**

\[ G_t = (N, E, T, \psi, \rho, \zeta) \]

\[ G(t) = (N(t), E(t)) \] **SNAPSHOT** at time \( t \in T \)

\[ N(t) = \{ x \in N : \psi(x, t) = 1 \} \]

\[ E(t) = \{ e \in E : \rho(e, t) = 1 \} \]

\[ G = (N, E) \] **FOOTPRINT** (underlying graph)

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Time-Varying Graph

$G_t$
Time-Varying Graph

$G_t$

past $\rightarrow$ present $\rightarrow$ future

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Time-Varying Graph

\[ G_t \]

- Known
- Unknown
- Past
- Present
- Future
Time-Varying Graph

Post Mortem

Off-line

Centralized

known

past
collected data to be analyzed

present

t

unknown

future

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Time-Varying Graph

unknown

t

future

present

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Time-Varying Graph

something must be known

t

future

present

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Time-Varying Graph

ASSUMPTIONS

a-priori knowledge
oracle

something must be known

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Time-Varying Graph: Common Assumption

FINITE FOOTPRINT \ G=(N,E)
Time-Varying Graph: Common Assumption

SYNCHRONOUS

Time is divided in rounds

Evolving graph, Temporal graph, Multi-layer (multiplex)

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Dynamic Networks

ASSUMPTIONS

DYNAMICS MODELS
(adversary)

- Temporal Connectivity
- 1-Interval Connectivity
- T-Interval Connectivity

- B. Haeuepler, F. Kuhn. *DISC* 2012
- D. Ilcinkas, A.M. Wade. *SIROCCO* 2013
- D. Ilcinkas, R. Klasing, A.M. Wade. *SIROCCO* 2014
- T. Erlerbach, M. Hoffmann, F. Kammer, ICALP 2015
1-Interval-Connectivity

SYNCHRONOUS

Time is divided in rounds

round
1-Interval-Connectivity

1-INTERVAL CONNECTED

Each $G(i)$ contains a spanning-tree $SPT(i)$ of $G$
1-Interval-Connectivity

1-INTERVAL CONNECTED

Each $G(i)$ contains a spanning-tree of $G$
T-Interval-Connectivity

**T-INTERVAL CONNECTED**

Each $G(i)$ contains a spanning-tree $SPT(i)$ of $G$

$SPT(i)$ persists for $T$ rounds: $i$, $i+1$, $i+2$, ..., $i+T-1$
Time-Varying Graph

TVG CLASSES

bounded recurrent
periodic

always connected

connected over time

c_{10} \rightarrow c_9 \rightarrow c_{11} \rightarrow c_{12} 

c_{13} \rightarrow c_6 \rightarrow c_5 \rightarrow c_4 \rightarrow c_{3} 

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Dynamic Networks: Algorithmic Results


Dynamic Networks: Algorithmic Results

- E. Coulouma, E Godard. “A characterization of dynamic networks where consensus is solvable”. SIROCCO 2013
Dynamic Networks: Algorithmic Results


And many more
Dynamic Networks: Algorithmic Results

Most of the Results

Some

Centralized

Decentralized

known

unknown

past

future

present

t

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Focus of this talk

Decentralized

unknown

present

future

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Focus of this talk

Decentralized

future

unknown

present

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The Model: Dynamic Rings

1-INTERVAL CONNECTED

at each round, the adversary can remove one link
the adversary is possibly unfair (a link might be removed forever)
The Model: Dynamic Rings

\( n \) nodes

\( k \) mobile agents

anonymous
silent
bounded memory
The Model: Dynamic Rings

n nodes

k mobile agents

anonymous
silent
bounded memory
local orientation

CHIRALITY

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The Model: Dynamic Rings

Each node has two distinct ports
When an agent arrives, it arrives at a port

The Model: Dynamic Rings
The Model: Dynamic Rings

If it decides not to move (e.g. wait), it goes in the center
The Model: Dynamic Rings

It does not know if the link is up or not!

When an agent wants to leave it moves to the **port**.
The Model: Dynamic Rings

If the link is there, it arrives at the incident node in the next round.

It does not know if the link is up or not!

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The Model: Dynamic Rings

It does not know if the link is up or not!

what if the edge is missing?
If the link is missing, it stays on the port until the next round

The Model: Dynamic Rings

It does not know if the link is up or not!
The Model: Dynamic Rings

FSYNC: all robots are activated in each round

LOOK-COMPUTE-MOVE

No communication (the agents are silent) !!!
The Model: Dynamic Rings

**FSYNC:** all robots are activated in each round

In a round

**LOOK-COMPUTE-MOVE**

See agents present at the node (center or on ports) and content of memory

Decide what to do (execute algorithm)

Possibly Move

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The Model: Dynamic Rings

**FSYNC**: all robots are activated in each round

In a round

**LOOK-****COMPUTE-****MOVE**

See agents present at the node (center or on ports) and content of memory

Decide what to do (execute algorithm)

Possibly Move

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The Model: Dynamic Rings

**FSYNC:** all robots are activated in each round

- **LOOK-COMPUTE-MOVE**
  - See agents present at the node (center or on ports) and content of memory
  - Decide what to do (execute algorithm)
  - Possibly Move

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Mobile Agents in Time-Varying Graphs

Has been studied only in **STATIC** graphs, and especially in the ring

E. Kranakis, D. Krizanc, E. Marcou
*The Mobile Agent Rendezvous Problem in the Ring*
Morgan & Claypool, 2010

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Mobile Agents in Time-Varying Graphs

RENDEZVOUS/GATHERING


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Gathering in Dynamic Rings

CROSS DETECTION

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Because of dynamics

T1

Strict Gathering is unsolvable in \((R, A)\); this holds regardless of chirality, cross detection, and knowledge of \(k\) and \(n\).
Because of dynamics

**T1**

Strict Gathering is **unsolvable** in \((R, A)\); this holds regardless of chirality, cross detection, and knowledge of \(k\) and \(n\).

Even without dynamics

**T2**

Gathering is **unsolvable** in \((R, A)\) if neither \(k\) nor \(n\) are known.

\(n\) and/or \(k\) must be known
**GATHERING: BASIC LIMITATIONS**

### Because of dynamics

**T1**

*Strict Gathering* is *unsolvable* in \((R, A)\); this holds regardless of chirality, cross detection, and knowledge of \(k\) and \(n\).

### Even without dynamics

**T2**

Gathering is *unsolvable* in \((R, A)\) if neither \(k\) nor \(n\) are known.

**T3**

If the homebases are not distinguishable, then Gathering is *unsolvable* in \((R, A)\); this holds regardless of chirality, cross detection, and knowledge of \(k\) and \(n\).
Gathering in Dynamic Rings

either $k$ or $n$ known

$\text{(R, A)}$

homebase

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Gathering in Dynamic Rings

$$(R, A)$$

$${\text{CONFIGURATION}}$$

$$C$$
Set of all possible configurations
Gathering in Dynamic Rings

$P$
Set of all periodic configurations

$C$
Set of all possible configurations

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GATHERING: BASIC LIMITATIONS

Even without dynamics

T4
Gathering is **unsolvable** in \((R, A)\) if \(C \in \mathcal{P}\); this holds regardless of chirality, cross detection, and knowledge of \(k\) and \(n\).

\[ C \text{ is not periodic} \]
Gathering in Dynamic Rings

Aperiodic configurations (the only feasible ones in static)
Gathering in Dynamic Rings

A

Aperiodic configurations (the only feasible ones in static)

Double palindromes

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Gathering in Dynamic Rings

\( \mathcal{A} \)
Aperiodic configurations
(the only feasible ones in static)

\( \mathcal{E} \)
Double palindromes with edge-edge axis of symmetry

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Gathering in Dynamic Rings

\[ A \]
Aperiodic configurations (the only feasible ones in static)

\[ \mathcal{E} \]
Double palindromes with edge-edge axis of symmetry

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GATHERING: BASIC LIMITATIONS

Even without dynamics

T4
Gathering is **unsolvable** in \((R, A)\) if \(C \in \mathcal{P}\); this holds regardless of chirality, cross detection, and knowledge of \(k\) and \(n\).

Because of dynamics

T5
**Without cross-detection and without chirality** Gathering is **unsolvable** in \((R, A)\) if \(C \in \mathcal{E}\); this holds regardless of knowledge of \(k\) and \(n\).
**GATHERING: FEASIBILITY**

<table>
<thead>
<tr>
<th>chirality</th>
<th>no chirality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$A$</td>
</tr>
<tr>
<td>$A$</td>
<td>$A \setminus E$</td>
</tr>
</tbody>
</table>

**With knowledge of n**

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GATHERING: FEASIBILITY

With knowledge of $n$

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GATHERING: FEASIBILITY

With knowledge of $n$

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GATHERING: **FEASIBILITY**

- **cross detection**
- **no cross detection**

\[ \mathcal{A} \]
\[ \mathcal{A} \setminus \mathcal{E} \]

**With knowledge of** \( n \)

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With knowledge of $n$, no cross-detection is possible if there is no chirality. However, with knowledge of $n$, the set $A - \mathcal{E}$ can be used to detect cross-detections.
## GATHERING: FEASIBILITY

<table>
<thead>
<tr>
<th>Chirality</th>
<th>No Chirality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Detection</td>
<td>$\mathcal{A}$</td>
</tr>
<tr>
<td>No Cross Detection</td>
<td>$\mathcal{A}$</td>
</tr>
</tbody>
</table>

**Knowledge of $n$ is more powerful**

**With knowledge of $k$ only**

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**GATHERING: FEASIBILITY**

<table>
<thead>
<tr>
<th>Chirality</th>
<th>No Chirality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Detection</td>
<td>No Cross Detection</td>
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<tr>
<td>$\mathcal{A}$</td>
<td>$\mathcal{A} \setminus \mathcal{E}$</td>
</tr>
<tr>
<td>$\mathcal{A}$</td>
<td>$\mathcal{A} \setminus \mathcal{E}$</td>
</tr>
</tbody>
</table>

*With knowledge of $k$ only*

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Different strategies depending on

- availability or lack of **cross detection**

- presence or absence of **chirality**
Two phases

Phase 1: The agents explore the ring
They might already solve Gathering. If so, they stop.
If not, the agents are able to elect a node or an edge and proceed to Phase 2

Phase 2: The agents gather
They try to gather around the elected node or edge.
If that is not possible (due to the ring dynamics), gathering occurs nevertheless at another place.
GATHERING: CROSS DETECTION - NO CHIRALITY

With knowledge of $n$

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With Cross Detection: Without Chirality

**Phase 1: Exploration**

- **Check-points**
- Move-left for 6n rounds
  
  I find out important global information and I act accordingly

- 0
- 6n
- 12n

If we have not gathered, I start Phase 2.

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Special Condition checked at round $6n$:

$P$: last time I met someone new going in my direction was less than $3n$ rounds ago; since then I traversed less than $n$ links.

$P$ true at round $6n$ means:

All agents moving in my direction form a single group; some may have not explored the whole ring; $P$ is true for all of them.

$P$ false at round $6n$ means:

All agents moving in my direction have explored the whole ring (hence they know $k$ and the configuration), and $P$ is false also for them.

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**With Cross Detection: Without Chirality**

**Phase 1: Exploration**

**Special Condition checked at round 6n:**

\( P: \) last time I met someone new going in my direction was less than \( 3n \) rounds ago; since then I traversed less than \( n \) links.

**If P is true, I continue in the same direction for 6n more rounds**

**If P is false, I switch direction and move for 6n more rounds**

During this time, I may TERMINATE if certain conditions occur

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With Cross Detection: Without Chirality

Phase 1: Exploration

Move-left for 6n rounds
Phase 1: Exploration

P: last time I met someone new going in my direction was less than $3n$ rounds ago; since then I traversed less than $n$ links.

With Cross Detection: Without Chirality

P false: everybody with my direction has explored

P true: everybody with my direction is here
Phase 1: Exploration

P: last time I met someone new going in my direction was less than 3n rounds ago; since then I traversed less than n links.
With Cross Detection: Without Chirality

Phase 1: Exploration

Switch
Keep-moving in the new direction for $6n$ rounds

Continue
Keep-moving left for $6n$ rounds
At round \(12n\): if there are \(k\) agents in this node AND crossed less than \(n\) links AND met someone less than \(9n\) rounds ago AND never met anybody in opposite direction: **TERMINATE**
Otherwise: **Phase 2**

**Switch**

**Continue**

At round \(12n\): if crossed less than \(n\) links and met someone less than \(9n\) rounds ago: **TERMINATE**
Otherwise: **Phase 2**

*Note that \(k\) agents in this node is not sufficient*
If an agent terminates in Phase 1, then all agents terminate and gathering has been correctly achieved. Otherwise, no agent terminates and all of them have done a complete tour of the ring.

Phase 2: Gathering

The agents know the configuration and know if gathering is feasible. If it is, they all elect the same leader (edge or node) and they start the phase moving towards it through the shortest path.
Gathering in Dynamic Rings

A
Aperiodic configuration
With Cross Detection: Without Chirality

**Phase 2: Gathering**

- **Check-points**
  - During this time I could detect termination in several ways
  - I check and act accordingly
  - Move-toward-leader.

- **12n**
  - If \( k \) of us are here TERMINATE

- **15n**
  - If I reached the leader, I become *ReachedElected* and switch direction

- **25n**
  - If I did not reach the leader I become *ReachingElected* and keep moving

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At round 15\textsubscript{n}, there is at most one group in state \textit{ReachingElected}, and at most two groups in state \textit{ReachedElected}. 
If there is an edge leader

there are two groups of agents in state \textit{ReachedElected} with opposite direction toward the \textit{ReachingElected} group
If there is a node leader

there is a unique group of agents in state \textit{ReachedElected}

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- the **ReachingElected** agents **switch direction** and try to reach the agents **ReachedElected** to join them

- the **ReachedElected** agents **keep same direction** and try to gather.
But the missing link can create several situations to be taken care of ...
**ReachedElected**

If I cross a group of agents, I switch direction to try to catch them.
- If they cross me again (double-crossing), TERMINATE
- If they do not cross me a second time (i.e., I join them) switch direction again and stay in *ReachedElected* state

**ReachingElected**

If I reach the leader: switch direction and become *ReachedElected*
- If I am blocked at a missing edge and I am reached by some other agent, I become *ReachedElected* and I keep my direction
- If I cross some other agent, I stop and wait.
  - if I meet anybody new while waiting in the next 2n rounds, switch direction and become *ReachedElected*; otherwise TERMINATE

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During this time I could detect termination in several ways.

In state *ReachedElected*:
- double-crossing a group of agents
With Cross Detection: Without Chirality

**Phase 2: Gathering**

**TERMINATE**

*In any state:*

**k agents on same node**

Gathering is achieved on this node

**blocked on a missing edge for** 2n **rounds**

If nobody reached us by now, the other group is on the other side of the edge and Gathering is achieved on this edge

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Phase 2 terminates correctly by round 25n.

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GATHERING: **COSTS**

<table>
<thead>
<tr>
<th>TIME</th>
<th>chirality</th>
<th>no chirality</th>
</tr>
</thead>
<tbody>
<tr>
<td>cross detection</td>
<td>$\mathcal{A}$</td>
<td>$\mathcal{A}$</td>
</tr>
<tr>
<td>no cross detection</td>
<td>$\mathcal{A}$</td>
<td>$\mathcal{A} \setminus \mathcal{E}$</td>
</tr>
</tbody>
</table>

With knowledge of $n$

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### GATHERING: COSTS

<table>
<thead>
<tr>
<th>TIME</th>
<th>chirality</th>
<th>no chirality</th>
</tr>
</thead>
<tbody>
<tr>
<td>cross detection</td>
<td>O(n)</td>
<td>O(n)</td>
</tr>
<tr>
<td>no cross detection</td>
<td>O(n log n)</td>
<td>O(n^2)</td>
</tr>
</tbody>
</table>

*With knowledge of n*

---

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Mobile Agents in Time-Varying Graphs

EXPLORATION

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Mobile Agents in Time-Varying Graphs

EXPLORATION

- C. Avin, M. Koucký, Z. Lotker. How to explore a fast-changing world (cover time of a simple random walk on evolving graphs). (ICALP 2008).


Mobile Agents in Time-Varying Graphs

EXPLORATION


- T. Erlebach, M. Hoffmann, F. Kammer On Temporal Graph Exploration. (ICALP 2015)

- M. Bournat, S. Dubois, and F. Petit, Computability of perpetual exploration in highly dynamic rings (ICDCS 2017)

Termination

Explicit Termination
all agents terminate knowing that the ring has been explored.

Partial Termination
at least one agent terminates knowing that the ring has been explored.
Main Questions:

Under what conditions is it possible to explore the dynamic ring?

When can the agents explicitly terminate?

What is the minimum number of agents necessary to explore?
Important factors influencing feasibility/termination

Chirality

Anonymity vs. presence of Landmark

Knowledge of exact size

Knowledge of bound on size

Level of synchronicity

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Semi-Synchronous (SSYNC)

Not all agents are activated at every round

zzz.. Every agent is activated infinitely often

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SSYNC

Not all agents are activated at every round
Not all agents are activated at every round

The agent might be sleeping next time the link appears ....
SSYNC

Not all agents are activated at every round

The agent might be sleeping next time the link appears ....

The link may be missing next time the agent is active ...
The agent might be sleeping next time the link appears ....

The link may be missing next time the agent is active ...

The agent may be sleeping every time it appears !!!
**SSYNC**

- **PT** - Passive Transport: as soon as the edge is present the agent moves (even if not active).
- **NS** - No Simultaneity: can move only when active and link is present.
- **ET** - Eventual Transport: the agent will be eventually active at a time when the link is present.

When activated, an agent finds itself on a port with a missing link.
SSYNC

**NS** - No Simultaneity: can move only when active and link is present

The agent may be sleeping every time it appears !!!

In NS exploration with any number of agents is impossible (even if if there is chirality, Knowledge of n, and a landmark)

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**SSYNC**

**ET** - Eventual Transport: the agent will be eventually active at a time when the link is present.
**SSYNC**

**ET** - Eventual Transport: the agent will be eventually active at a time when the link is present
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PT Passive Transport: as soon as the edge is present the agent moves (even if not active).
**SSYNC**

**PT** - Passive Transport: as soon as the edge is present the agent moves (even if not active).
SSYNC – Passive Transport (PT) - Impossibilities

Explicit Termination of 2 agents is impossible (even with chirality, knowledge of n and a landmark)

Without chirality, exploration with 2 agents is impossible (even if n is known and there is a landmark)
SSYNC – Passive Transport (PT) - Impossibilities

Explicit Termination of 2 agents is impossible (even with chirality, knowledge of n and a landmark)

Without chirality, exploration with 2 agents is impossible (even if n is known and there is a landmark)

Note that, even without dynamics:
Without an Upper Bound and without landmark, exploration with 2 agents is impossible (even if there is chirality)
### SSYNC – Passive Transport (PT) – Possibility results

<table>
<thead>
<tr>
<th></th>
<th>Chirality &amp; Landmark</th>
<th>(O(n^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Known Bound N</td>
<td>Partial termination</td>
</tr>
</tbody>
</table>

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### SSYNC – Passive Transport (PT) – Possibility results

<table>
<thead>
<tr>
<th>2</th>
<th>Chirality Known Bound N</th>
<th>O(N^2) Partial termination</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Chirality &amp; Landmark</td>
<td>O(n^2) Partial termination</td>
</tr>
</tbody>
</table>

- **Necessary**:
- **Necessary without Landmark**: \(\Omega(n \ N)\)
- **Explicit Termination impossible**: \(\Omega(n^2)\)

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### SSYNC – Passive Transport (PT) – Possibility results

<table>
<thead>
<tr>
<th>Necessary</th>
<th>Necessary without Landmark</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="ssync-passive-transport-possibility-results.png" alt="Image" /></td>
<td><img src="ssync-passive-transport-possibility-results.png" alt="Image" /></td>
</tr>
</tbody>
</table>

- **Chirality Known Bound N**
  - Partial termination
  - $O(N^2)$
- **Chirality & Landmark**
  - Partial termination
  - $O(n^2)$
- **Known Bound N**
  - Partial termination
  - $O(N^2)$
- **Landmark**
  - Partial termination
  - $O(n^2)$
SSYNC – with Chirality and Known Upper Bound N

Assumptions

<table>
<thead>
<tr>
<th>2 agents</th>
<th>Upper Bound N</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSYNC- PT anonymous</td>
<td>Chirality</td>
</tr>
</tbody>
</table>

Complexity

<table>
<thead>
<tr>
<th>O(N^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega(n N) ) is a Lower Bound</td>
</tr>
</tbody>
</table>

Partial termination

Without an Upper Bound, exploration with 2 agents of an anonymous ring is impossible (even if there is chirality) **even without dynamics**

Without chirality, exploration with 2 agents is impossible (even with an Upper Bound) **because of dynamics**

Explicit Termination is impossible
SSYNC – with Chirality and Known Upper Bound N

<table>
<thead>
<tr>
<th>2 PT</th>
<th>Chirality Upper Bound N</th>
<th>$O(N^2)$ Partial termination</th>
</tr>
</thead>
</table>

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SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

Moving left: either in state INIT or in state REVERSE

Moving right: always in state BOUNCE
SSYNC – with Chirality and Known Upper Bound $N$

**ZIG-ZAG**

**BOUNCE** when catching the other agent waiting at a missing link

*Left-to-right direction*
SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

BOUNCE when catching the other agent waiting at a missing link

Left-to-right direction
SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

BOUNCE when catching the other agent waiting at a missing link

Left-to-right direction

REVERSE when finding an empty missing link

Right-to-left direction

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SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

BOUNCE when catching the other agent waiting at a missing link

Left-to-right direction

REVERSE when finding an empty missing link

Right-to-left direction

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SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

**GO-LEFT**

If find a blocked edge with the other agent waiting in the left port, become **BOUNCE**, switch direction and starts moving right.

If, in state **BOUNCE**, find a missing edge before having traversed N edges, switch direction and become **REVERSE** and continue

**TERMINATION CONDITIONS**

1) Discovering to have traversed N consecutive edges in the same direction:

2) Catching the other agent at a distance smaller than the one of the previous catch
**SSYNC – with Chirality and Known Upper Bound N**

**ZIG-ZAG**
- A **REVERSE** (or Init) agent **Bounces** when it catches the other agent moving left.
- A **BOUNCE** agent **Reverses** when it finds a missing link moving right.

The distance traveled left by an agent to catch the other agent **keeps increasing**

**Except** when the ring has been already explored, in which case **it may decrease**

An agent terminates in two ways:
1) after visiting $N$ nodes (either in BOUNCE or REVERSE mode)
2) when noticing **such a decrease**
SSYNC – with Chirality and Known Upper Bound $N$

Number of moves: $O(N^2)$

$\Omega(n \cdot N)$ is a Lower Bound

Partial Termination

Theorem

In SSYNC, with chirality and knowledge of an upper bound on the ring size, the ring can be explored with partial termination in $O(N^2)$ rounds.
SSYNC – with Chirality and Known Upper Bound N

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 agents SSYNC- PT anonymous</td>
<td>$O(N^2)$ Partial termination</td>
</tr>
</tbody>
</table>

Without an Upper Bound, exploration with 2 agents of an anonymous ring is impossible (even if there is chirality)

Without chirality, exploration with 2 agents is impossible (even with an Upper Bound)

Explicit Termination is impossible

Even without dynamics

Because of dynamics

Ω(n N ) is a Lower Bound

- Prague 2018
SSYNC – with Chirality and Known Upper Bound N

<table>
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<td>$O(N^2)$</td>
</tr>
<tr>
<td>Upper Bound N</td>
<td>$\Omega(n^2)$ is a Lower Bound</td>
</tr>
<tr>
<td>Landmark Chirality</td>
<td></td>
</tr>
</tbody>
</table>

**Partial termination**

Without an Upper Bound, exploration with 2 agents of an anonymous ring is impossible (even if there is chirality)

Even without dynamics

Without chirality, exploration with 2 agents is impossible (even with an Upper Bound)

Because of dynamics

Explicit Termination is impossible
## SSYNC – Possibility results

<table>
<thead>
<tr>
<th>Agents</th>
<th>Assumptions</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Chirality &amp; Known Bound N</td>
<td>$O(N^2)$ Partial termination</td>
</tr>
<tr>
<td>2</td>
<td>Chirality &amp; Landmark</td>
<td>$O(n^2)$ Partial termination</td>
</tr>
<tr>
<td>3</td>
<td>Known Bound N</td>
<td>$O(N^2)$ Partial termination</td>
</tr>
<tr>
<td>3</td>
<td>Landmark</td>
<td>$O(n^2)$ Partial termination</td>
</tr>
<tr>
<td><strong>ET</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Chirality</td>
<td>Unconscious exploration</td>
</tr>
<tr>
<td>3</td>
<td>Known n</td>
<td>Finite number of moves Partial termination</td>
</tr>
</tbody>
</table>
### SSYNC – Impossibility results

<table>
<thead>
<tr>
<th>Agents</th>
<th>Assumptions</th>
<th>Even if</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>Any</td>
<td>None</td>
<td>Chirality, Known n, Landmark, distinct Ids</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>No chirality, anonymous</td>
<td>Known n, Landmark</td>
</tr>
<tr>
<td>PT</td>
<td>2</td>
<td>None</td>
<td>Chirality, known n, Landmark</td>
</tr>
<tr>
<td>ET</td>
<td>Any</td>
<td>Landmark</td>
<td>Known bound N, Chirality, Landmark, Distinct Ids</td>
</tr>
</tbody>
</table>
### FSYNC

<table>
<thead>
<tr>
<th>Agents</th>
<th>Assumptions</th>
<th>Even if</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Size unknown, No landmark</td>
<td>Non-anonymous, Chirality</td>
<td>Termination Impossible</td>
</tr>
<tr>
<td>Any</td>
<td>Size unknown, No landmark, Anonymous</td>
<td>Chirality</td>
<td>Termination Impossible</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Agents</th>
<th>Assumptions</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Known Bound N</td>
<td>$3N-6$</td>
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<tr>
<td></td>
<td></td>
<td>Explicit termination</td>
</tr>
<tr>
<td>2</td>
<td>Chirality and Landmark</td>
<td>$O(n)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Explicit termination</td>
</tr>
<tr>
<td>2</td>
<td>Landmark</td>
<td>$O(n \log n)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Explicit termination</td>
</tr>
</tbody>
</table>

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Improve the time bounds **without cross detection**

- **chirality**: $O(n \log n)$
- **no chirality**: $O(n^2)$

Gathering in **other dynamic graphs**

Gathering with **different dynamics**

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Small gaps between upper and lower bounds

Exploration of other dynamic graphs

Exploration with different dynamics
GENERAL CONCLUDING OBSERVATIONS

VERY LITTLE IS KNOWN
There is still a lot to discover
The End
Thank you
Questions ?
Thank you