LECTURE 6
PUBLIC GOODS: LINDAHL EQUILIBRIUM

Public Economics II, Epiphany Term 2003

Aim of Lecture 6: ● Introduce Lindahl Pricing
● Derive the equilibrium under Lindahl Pricing
● Show that the Lindahl Equilibrium is Pareto Efficient (i.e. that the Samuelson rule holds).

6.1 Introduction

For efficiency in private-goods consumption all marginal rates of substitutions must equal the respective relative price. Since all individuals in competitive equilibrium face the same relative prices, the implication is that all marginal rates of substitutions are equalised across individuals. This in turn implies that different individuals would consume different quantities of the various goods. However, when we introduce a public good, all individuals have to consume the same amount of it. Then, all individuals cannot equalise their marginal rates of substitutions to the common relative price. One may suspect that if individuals faced different (personalised) prices, one could equalise marginal rates of substitution across individuals, even though they consume the same amount of the public good. This is the idea behind Lindahl pricing: Lindahl prices are personalised prices.

6.2 Economy with Lindahl Prices

As in Lecture 5, we will for simplicity consider 2 households, one private good, $x$, and one public good, $G$.

Utilities

$U^h = U^h(x^h, G)$

for $h=1,2$.

Budgets

$x^h + \tau^h pG = \omega^h$

for $h=1,2$.

Aggregate production

We use for simplicity the production technology specified in Example 2, Lecture 5:

$G = \theta(\omega^1 + \omega^2 - x^1 - x^2)$

N.B. In equilibrium with this technology the producer price $p$ must equal the technology coefficient $\theta$. 
6.3 Lindahl Equilibrium

6.3.1 Individuals’ optimisation problems

For $h=1,2$: \[
\max U^h(x^h, G) \\
\text{s.t. } x^h + \tau^h pG = \omega^h
\] \[\iff \max U^h(\omega^h - \tau^h pG, G)\]

First-order condition: \[-U^h_x \tau^h p + U^h_G = 0 \iff \frac{U^h_G}{U^h_x} = \tau^h p \] (1)

Thus, the marginal rate of substitution between public and private good equals the (personalised) relative price $\tau^h p$.

One can solve for the demand of the public good by individual $h$, which will be a function of the price and endowment: \[G^h = L^h(\tau^h p, \omega^h)\]

for $h=1,2$.

Exercise 6.1

Let $U^h = \ln(x^h) + \frac{\eta^h}{1-\eta^h} \ln(G)$. Solve for the individuals’ demand functions.

6.3.2 Equilibrium

The Lindahl Equilibrium is a pair of cost shares $\{\hat{\tau}^1, \hat{\tau}^2\}$ and public goods provision $G^*$ such that \[\hat{\tau}^1 + \hat{\tau}^2 = 1 \] (2)
\[G^* = L^1(\hat{\tau}^1 p, \omega^1) = L^2(\hat{\tau}^2 p, \omega^2) \] (3)

N.B. ^ and * denote equilibrium levels.
Exercise 6.2

Let \( U^h = \ln(x^h) + [\eta^h/(1-\eta^h)]\ln(G) \). Solve for the Lindahl equilibrium (i.e. find \( \tau^1, \tau^2, G^* \)).

6.3.3 Efficiency

Exercise 6.3

Show that the Lindahl Equilibrium (equations 2-3) is Pareto Efficient. Hint: Compare with the Samuelson Rule.

6.3.4 Other properties

If the public good is a normal good then (everything else equal) an individual with a higher endowment, \( \omega^h \), (a richer individual) will face a higher Lindahl price. The reason is that a richer person would tend to spend more on public goods if the public good is normal (due to the income effect). However, in equilibrium everybody must consume the same amount of the public good. The only way to accomplish that is to make the richer individuals to reduce their demand for the public good by asking a higher price of them.

Exercise 6.4

Verify this property of the Lindahl equilibrium for the utility function in Exercise 6.2 [this utility function guarantees normality of consumption goods because it is additively separable]. Suggestion: Proceed from your solution to Exercise 6.2.

Exercise 6.5

This time, let the utility function be quasi-linear, and same for all individuals:

\[
U^h = x^h + \phi(G)
\]

where \( \phi'(G)>0, \phi''(G)<0 \).

Show that the Lindahl price is the same for all individuals (in particular equals \( p/n \), where \( n \) is the number of individuals in the economy). What is the reason for this result?

6.4 Implementing Equilibrium

Implementing the Lindahl equilibrium will be difficult, because it relies on personalised prices. The prices depend on personal characteristics (the endowments and the preferences). Individuals would then have to report their preferences and endowments and this creates an incentive problem. Individuals would have an incentive to misreport.